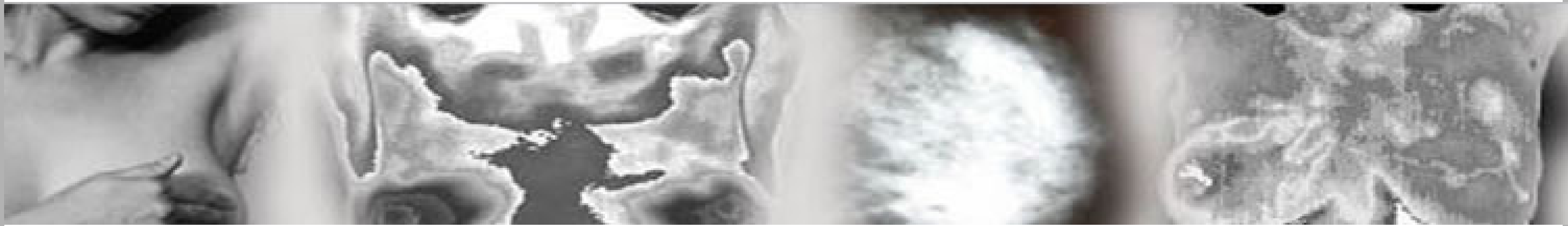

USING WAVELETS ON DENOISING INFRARED MEDICAL IMAGES DATA BASE

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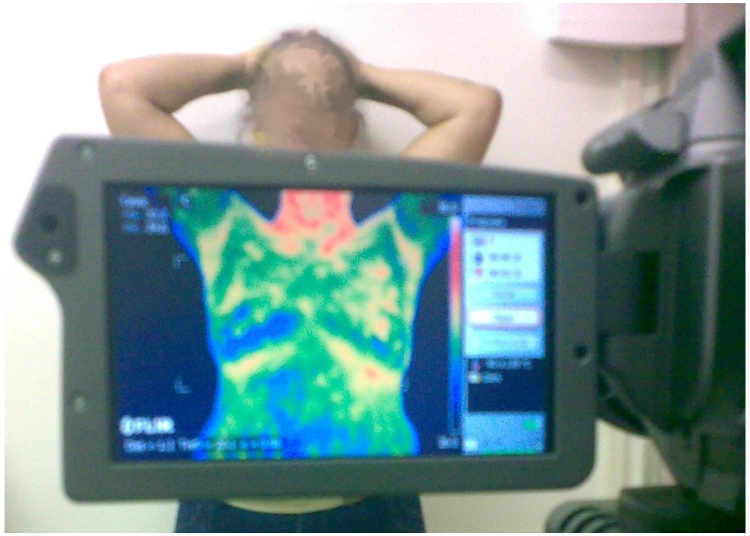
Introduction

- Computed aided diagnostic - CADx system makes substantial use of image processing and a great amount of data => efficient **contend based retrieval from image database**

- Image **restoration** after storage and transition is fundamental for the quality of the other stages in the image processing.

- Studies showed that infrared (IR) based image analysis could **identify breast modifications earlier** than others exams.

- To be efficiently implemented, CADx must first consider a great number of patients followed by years; maintain record and comparison with others types of diagnoses, combine and integrate data to allow **mining** possible conclusions system.



Thermograms are acquired by a thermographic camera that is sensitive to infrared IR.

It is a physiological examination and is 50x cheaper than the mammogram.

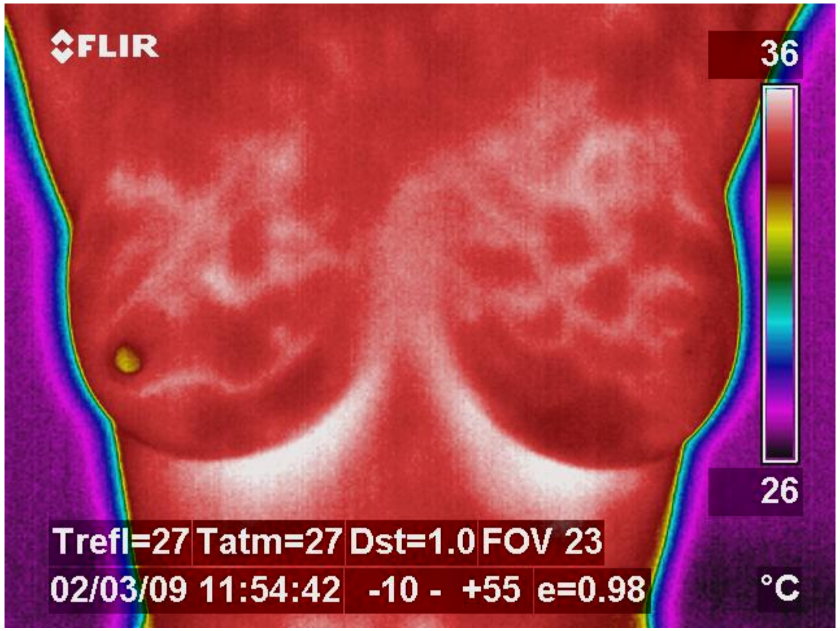
IR has potential to detect breast cancer 10 years earlier than the nowadays traditionally golden method.

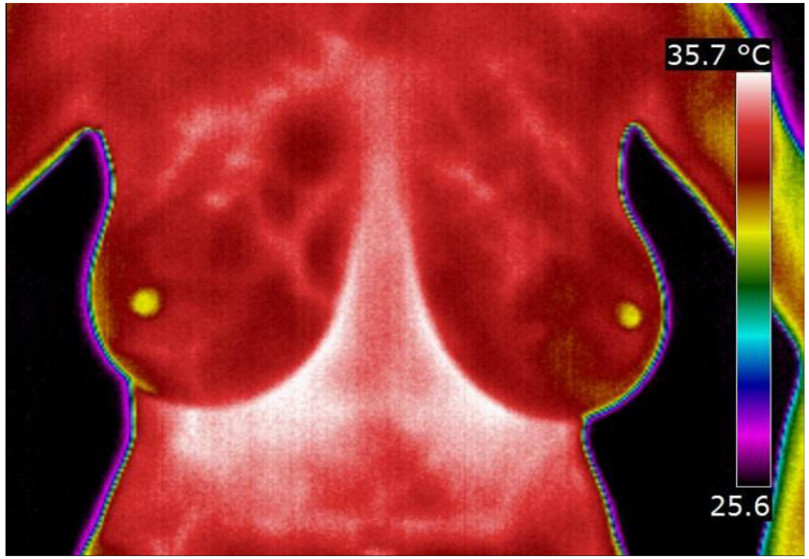
It can also be used for diagnosis of **young women's** tumours (young breasts present dense tissues that makes **difficult early detection** of pathologies by the X-ray).

IR do not use ionizing radiation, venous access (or others invasive procedures), is painless and do not touch the patient.

The problem is the absence of CAD systems to aid the such diagnosis.

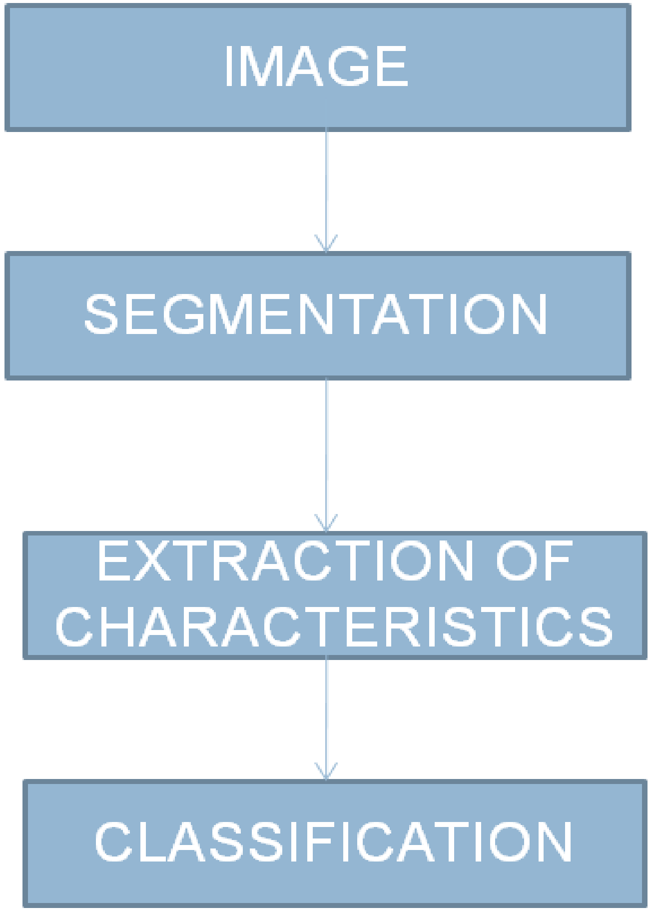
***Retroareolar* Carcinoma**





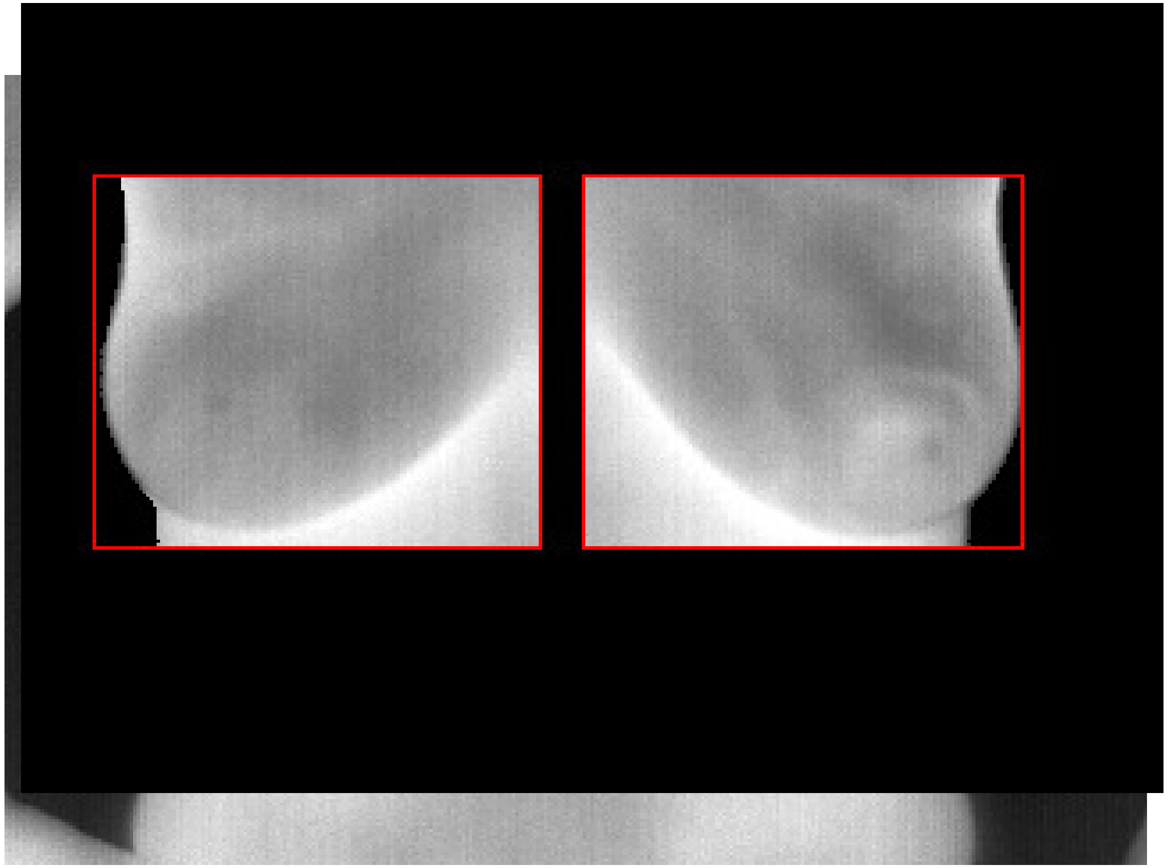
You can see that this is a normal breast (very symmetrical !)
but how to make the computer "see" the same?

CADx Pipeline

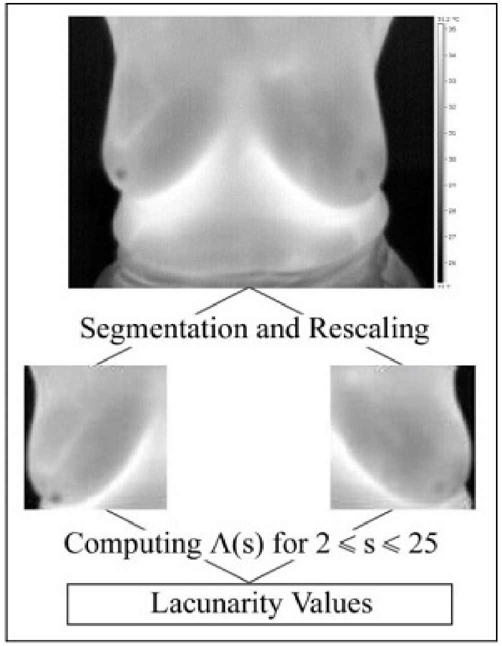


- thermal images
- automatic segment two regions of interest (ROI): the right breast and the left one;
- Then the extraction of the features;
- The last step is the classification of the patient image.

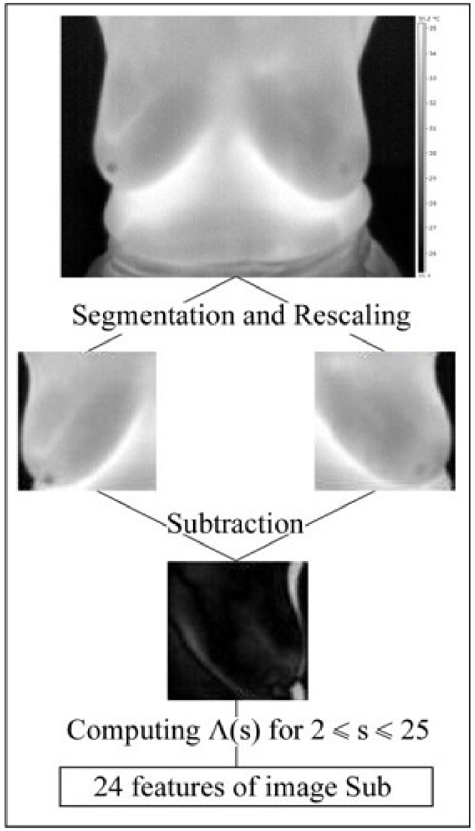
The project on development:



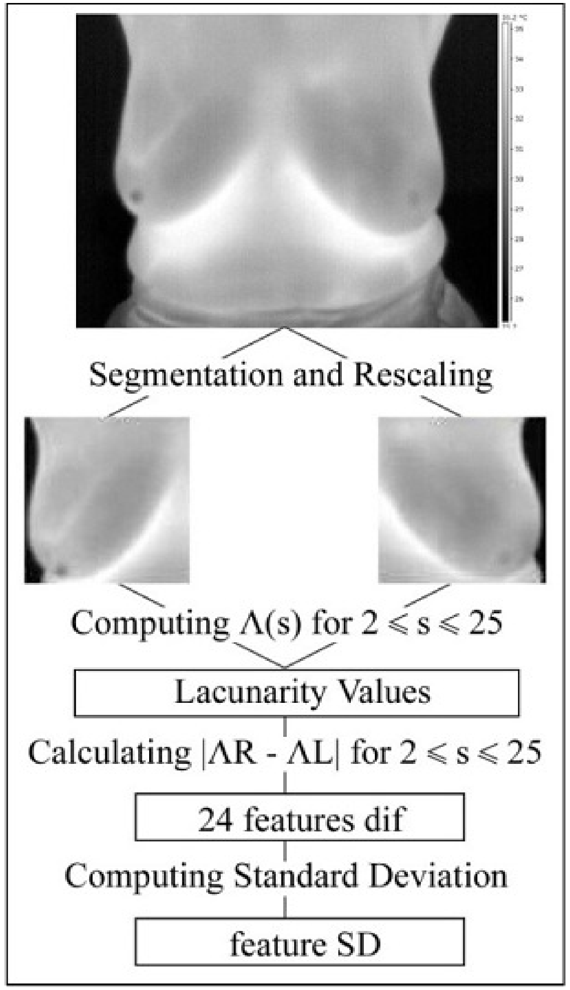
Features Extraction –> Classification



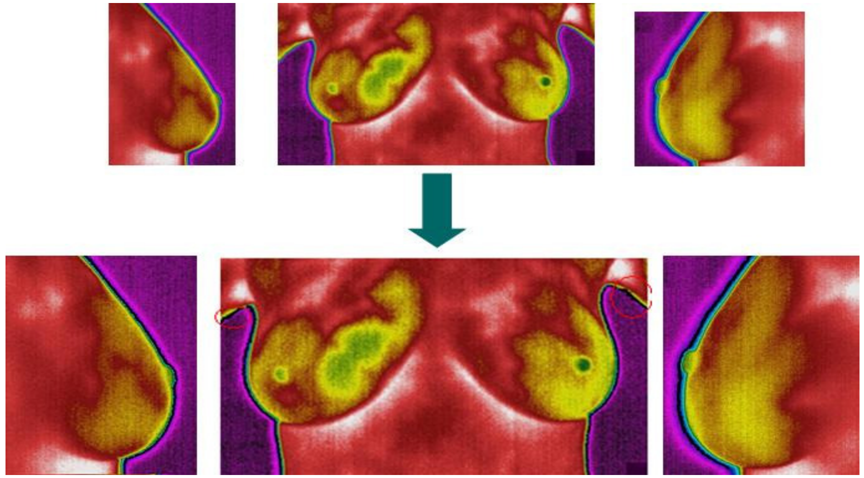
48 features



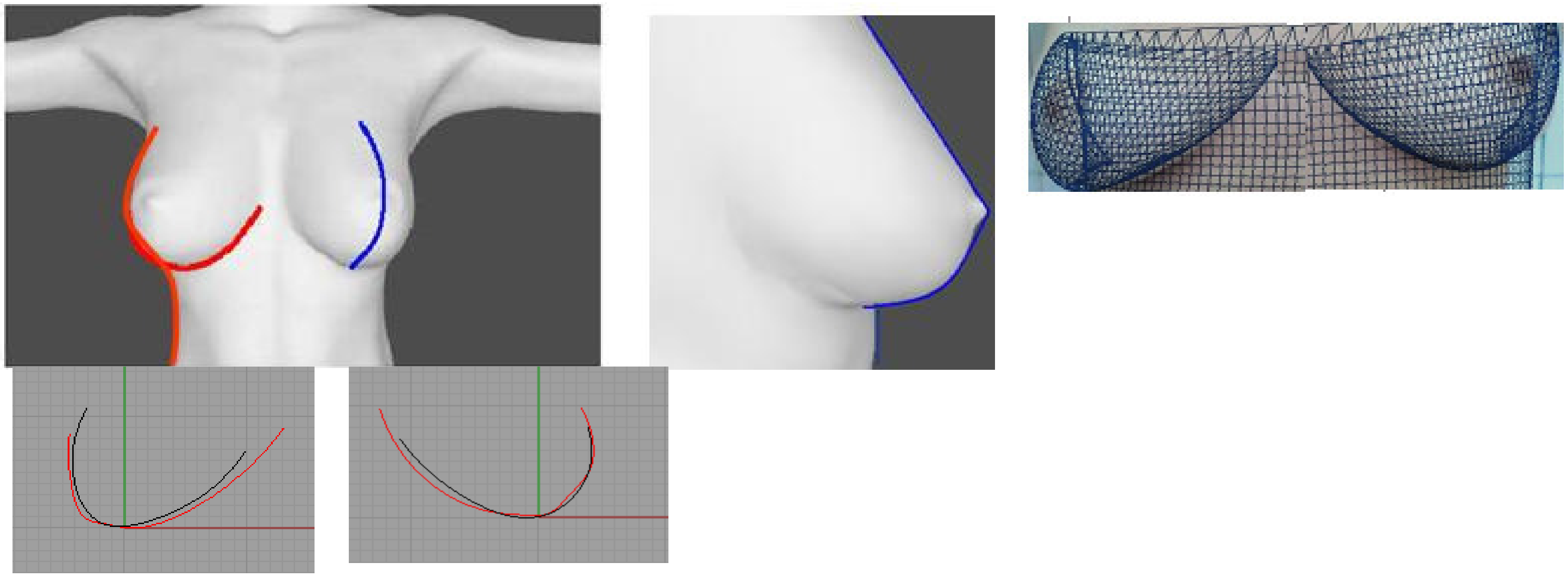
24 features



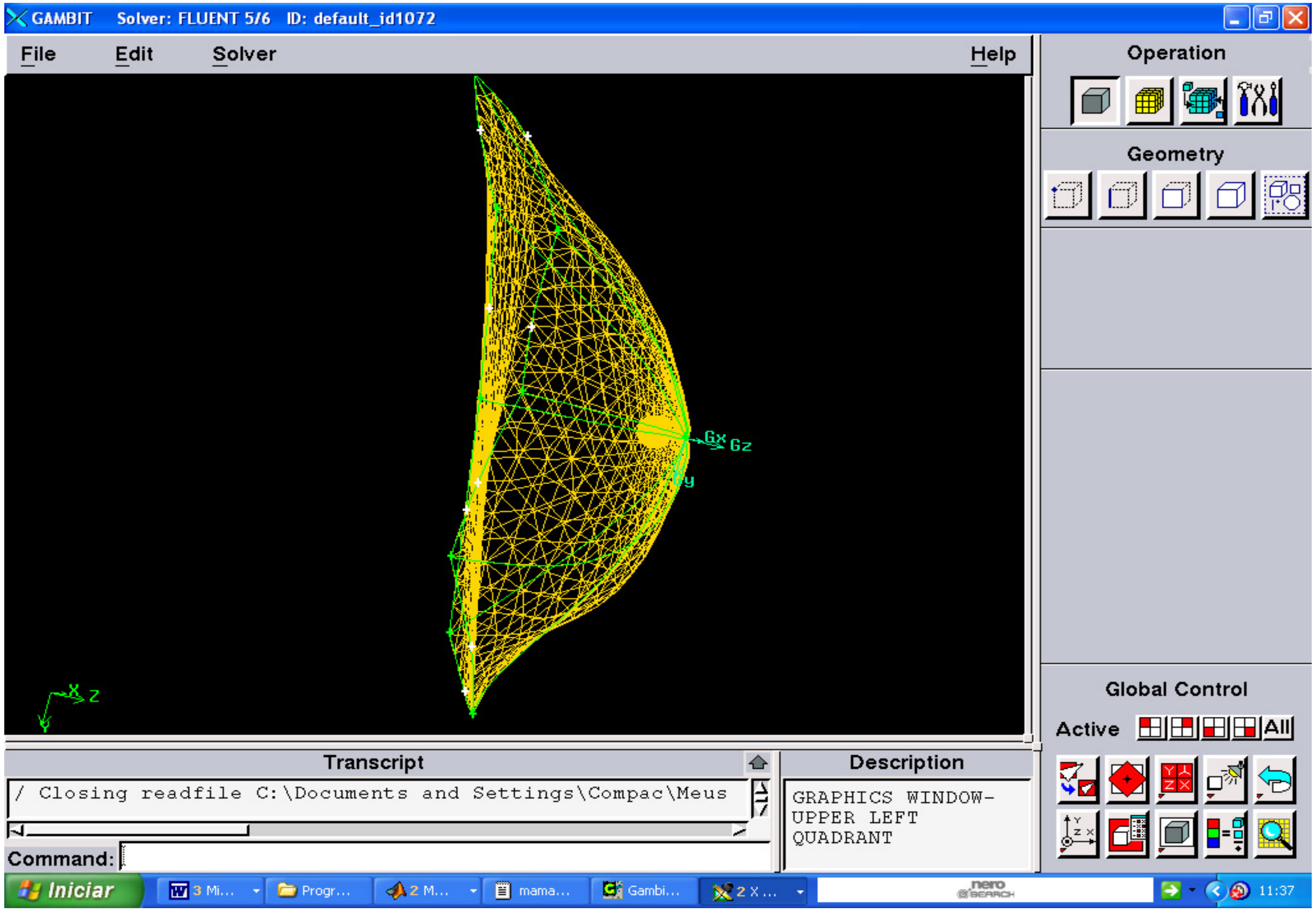
AI and techniques of machine machine learning as used for classification



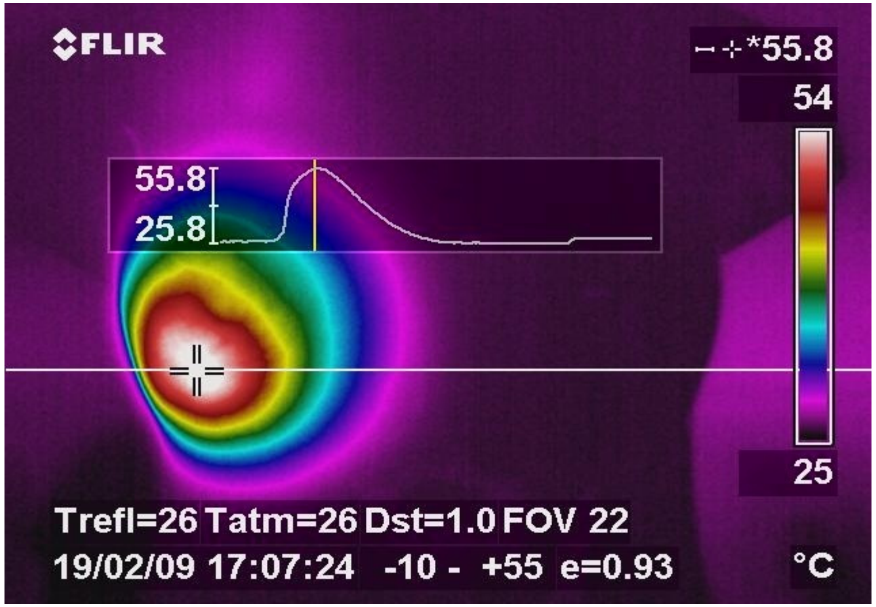
Breast reconstruction by IR images



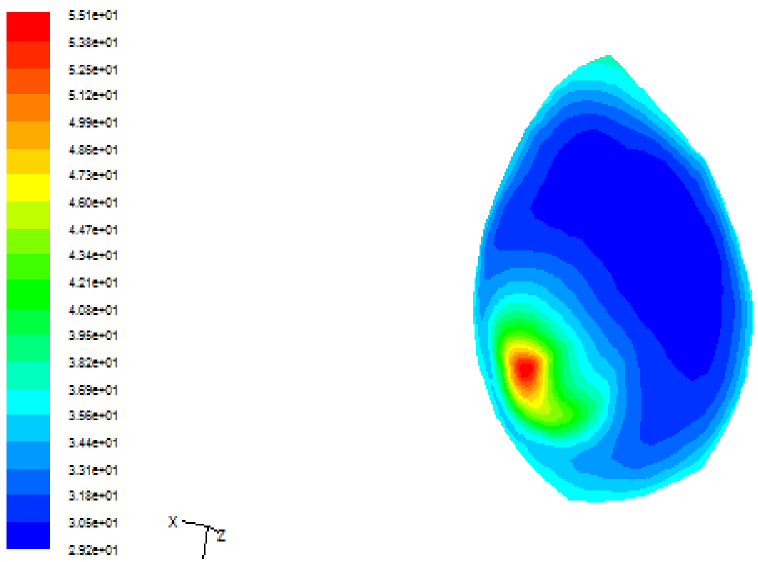
Finite Volume Method for 3D modeling



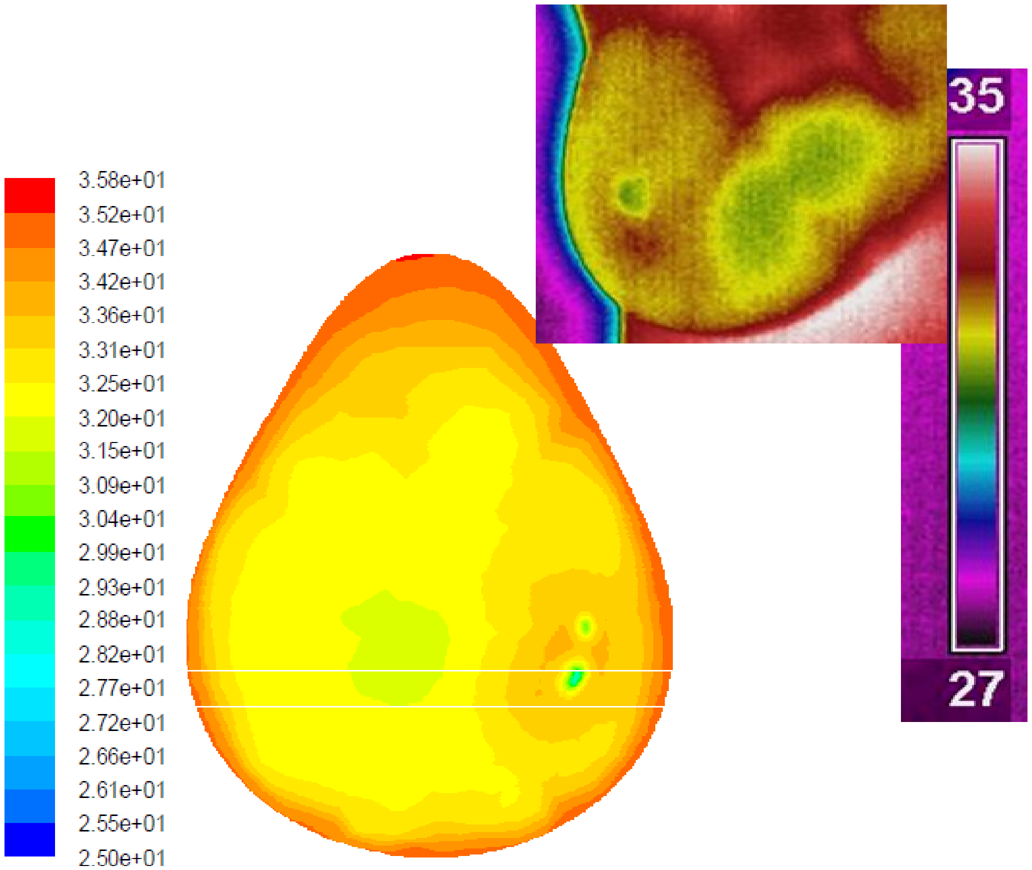
Experimental
x
Numerical



- The depth and the radius of a hypothetical tumor are considered to calculate breast temperatures through parametric analysis



Comparison between the temperature profiles of the thermogram and the surrogate breast meshed with 13748 nodes



Main objective:

- Best discrete wavelet (DW) scheme for
  - denoise ,
  - storage and
  - retrieval

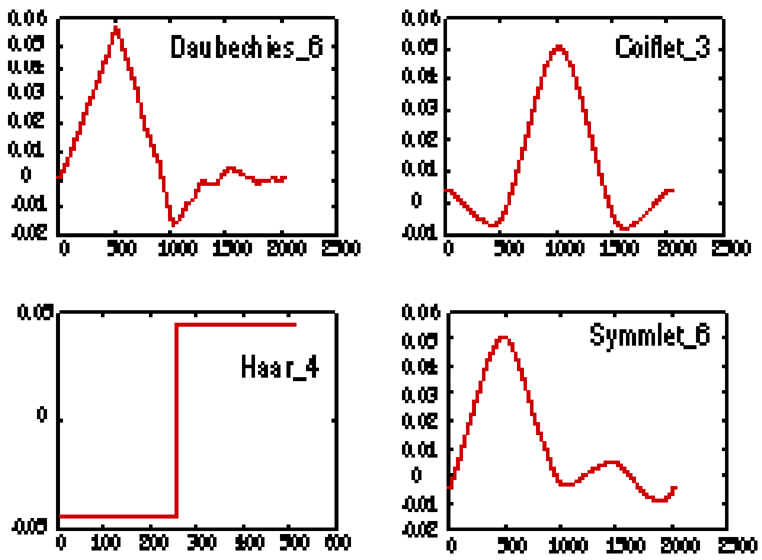
  for the project of an infrared image database to aid breast disease diagnostic in a tropical climate country

  ProEng project:

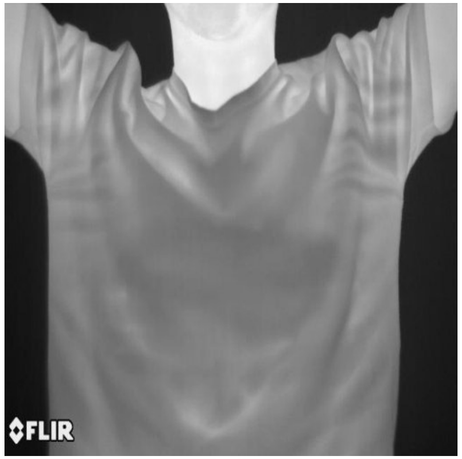
  http://tvbrasil.ebc.com.br/reporterbrasil/video/31312/

First part

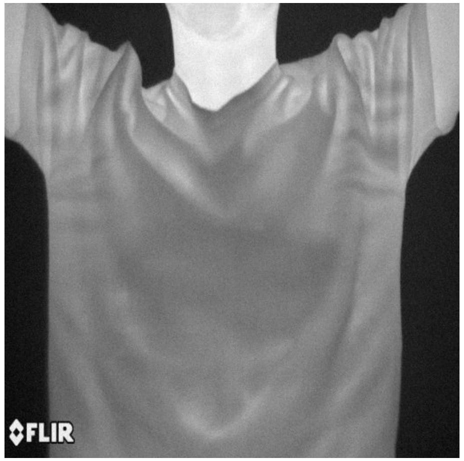
- Results and conclusions of an experimental study that intent to find the best family of wavelets to reduce noise of medium resolution infrared images.



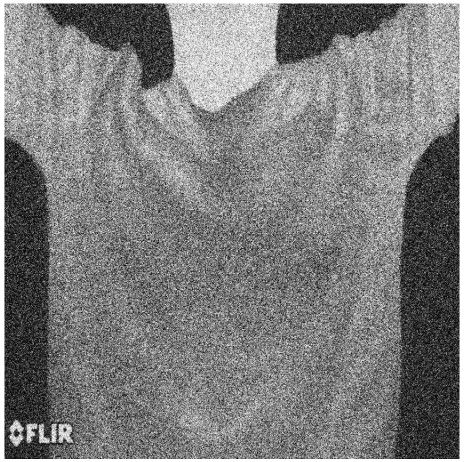
8 different real images + noise



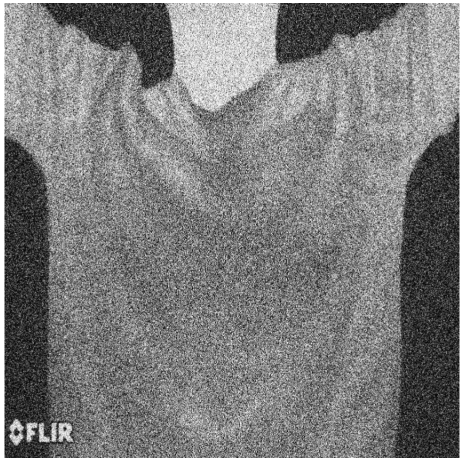
Original



Low noise



High



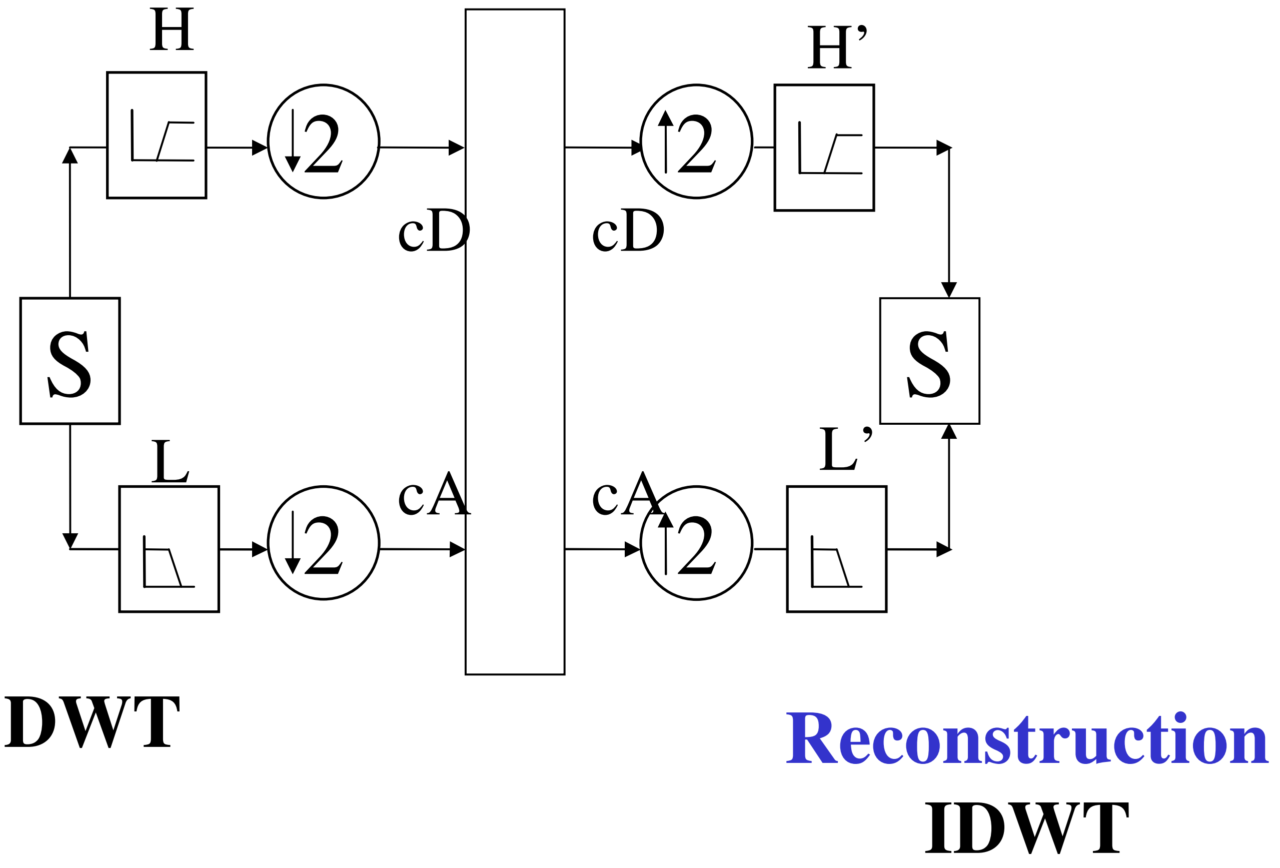
Medium

resolution:
640x 480

3 degradation levels
Additive White Gaussian Noise (AWGN):
$\sigma = 5$,
$\sigma = 25$,
and
$\sigma = 50$

Total: 32 images of same type separated on 4 groups concerning the level of noise (0, 5, 25 and 50).

Low and High pass filters



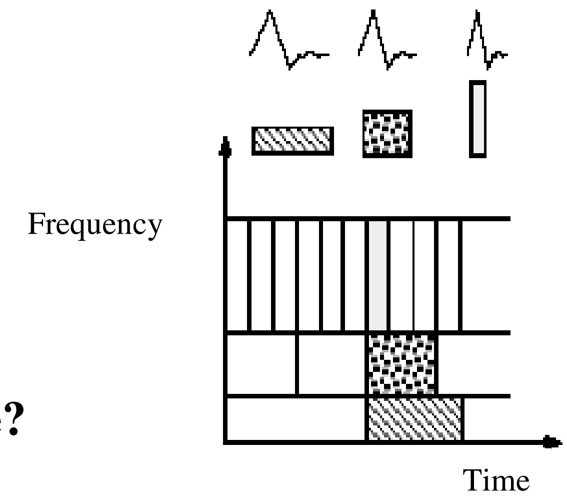
Discrete wavelet transforms (DWT) is very effective in analyzing images because it at same time

reduce the storage,

improve the image quality and

promote content based retrieval of the data.

What is the best wavelet approach to be used in a project of an **image database for medium resolution infrared images** in screening of breast diseases



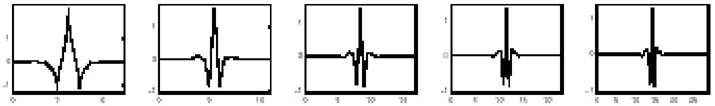
What base?

How denoise?

Wavelet ?

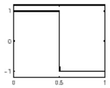
Wavelet

Coiflets

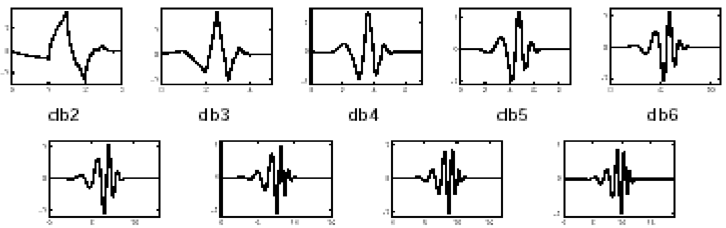


coif1 coif2 coif3 coif4 coif5

Haar

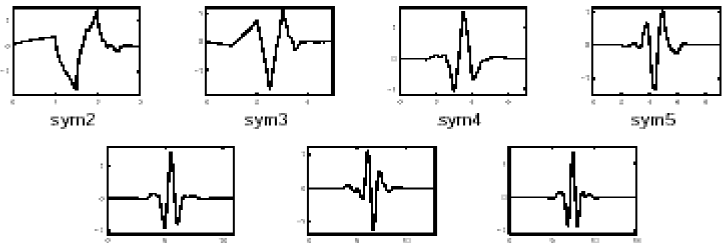


Daubechies



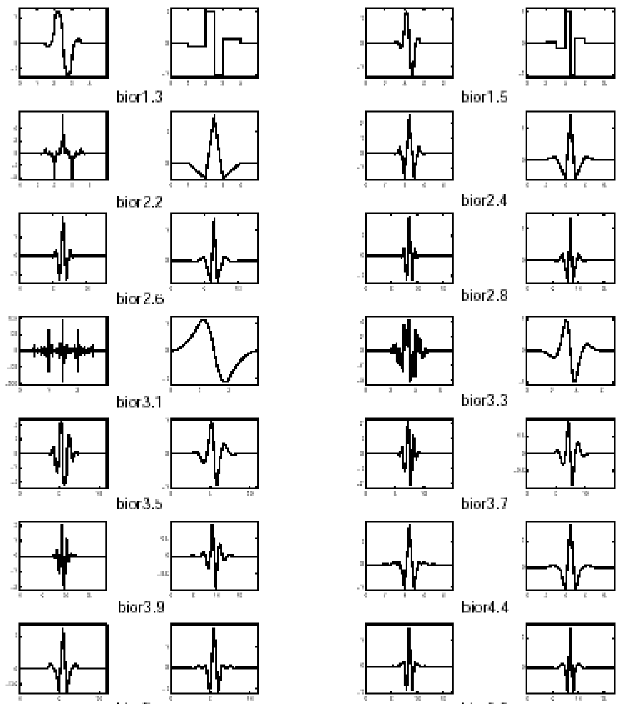
db2 db3 db4 db5 db6
db7 db8 db9 db10

Symlets



sym2 sym3 sym4 sym5
sym6 sym7 sym8

Biortogonal



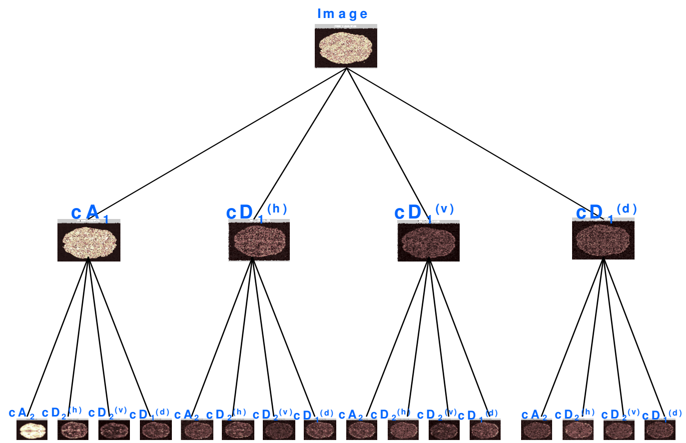
Types experimented (with various vanishing moments)

- Biorthogonal: 1.1 to 6.8
- Coiflets, 1 to 5
- Daubechies, 2 to 45
- Haar,
- Meyer,
- Reverse Biorthogonal: 1.1 to 6.8
- Symmlets - 2 to 28

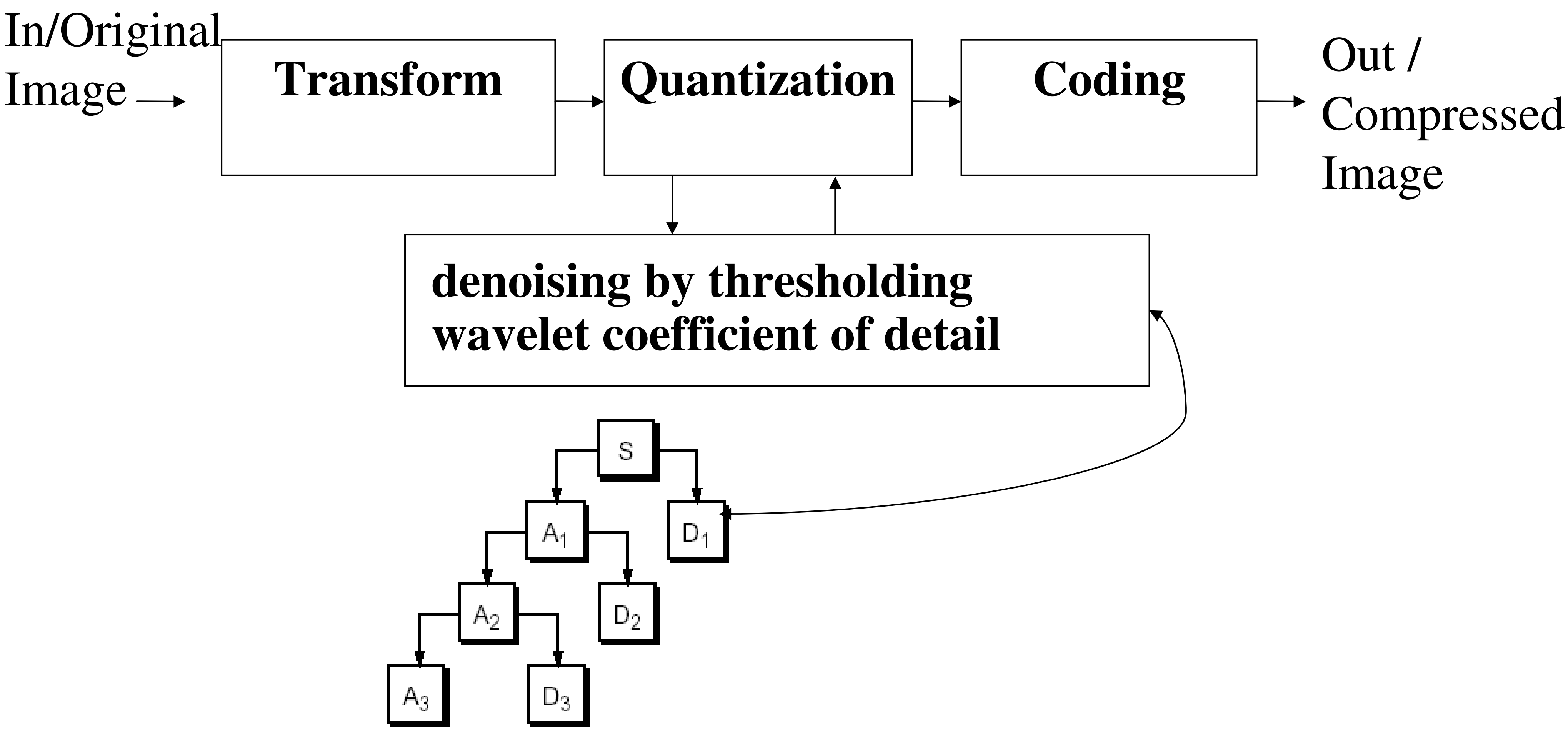
composing a total of 108 different variations!

Generic denoising procedures by DWT involve three steps:

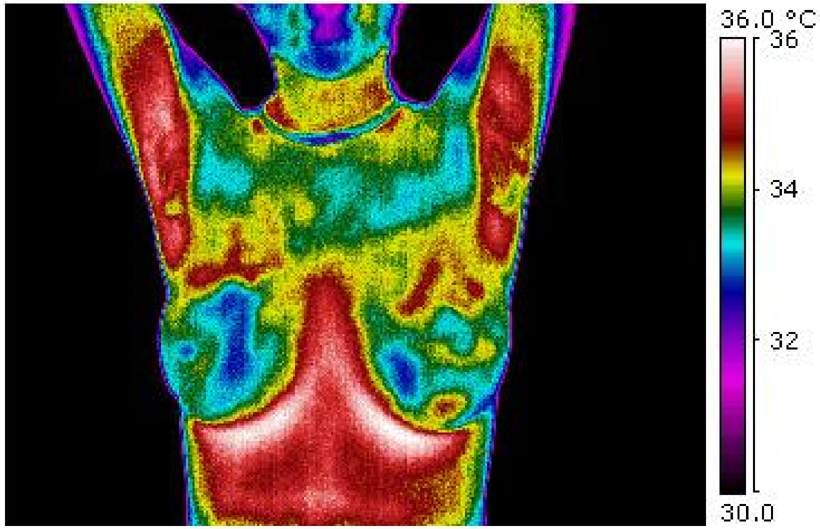
- wavelet decomposition,
- threshold of coefficients related to noise in the wavelet domain , and
- reconstruction by inverse wavelet transform into the spatial domain

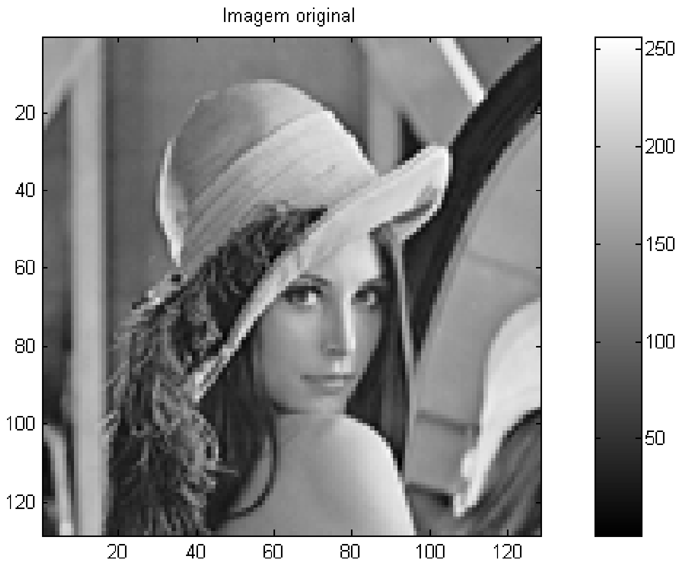


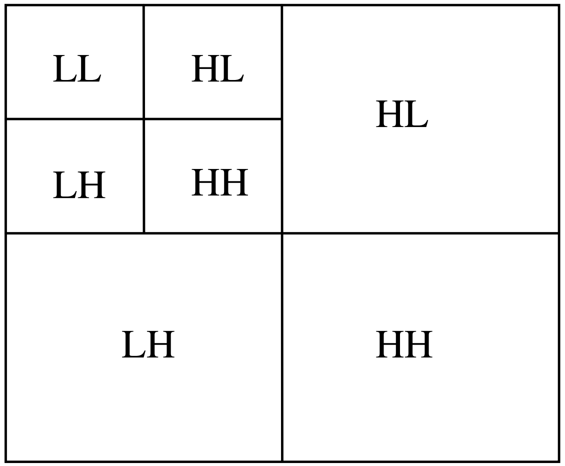
Denoising


In/Original Image
Transform
Quantization
Coding
Out / Compressed Image
denoising by thresholding wavelet coefficient of detail
S
A1
D1
A2
D2
A3
D3

Decomposition
&
Denoising







wavelet decomposition step

- an image is decomposed into a sequence of spatial resolution images using DWT.
- In these, a given **j** level of decomposition can be performed resulting in **3j+1** different frequency bands of low (**L**) and high (**H**) components of the original image, namely, **LLj** , **LHj**, **HLj** and **HHj**

Wavelet **denoising**

- Identify **low** and **high** energy coefficients
- Modify noisy coefficients by **adaptive thresholding**
- We use the **optimal reconstruction threshold:**

$$T = \sigma_n{}^2 / \sigma$$

σ_n^2 = Noise variance

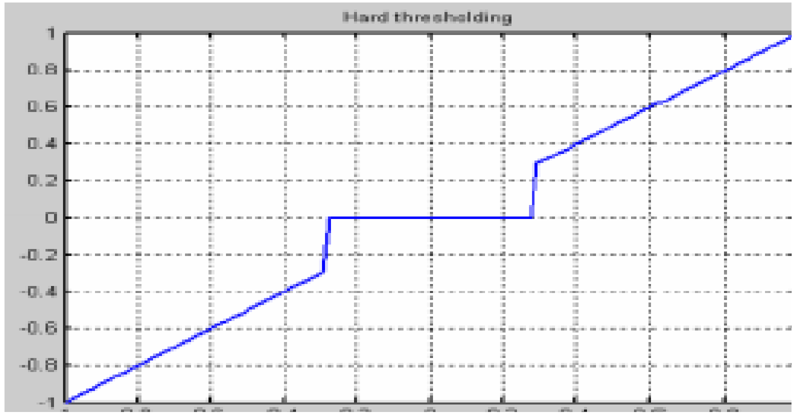
σ = Original Signal variance

(and Hard & soft Thresholding approach)

Setting to zero value of coefficients which are considered negligible.

$$y_{soft}(t) = \begin{cases} \operatorname{sgn}(x(t)) \cdot (|x(t) - \delta|), & |x(t)| > \delta \\ 0, & |x(t)| < \delta \end{cases}$$

$$y_{hard}(t) = \begin{cases} x(t), & |x(t)| > \delta \\ 0, & |x(t)| < \delta \end{cases}$$



where δ is the threshold value, and
sgn( ) is the *signal function* (it results +1 when the argument is up to zero and -1 otherwise).

The thresholding method proposed is not based *on a unique value* δ for threshold but testing all possibilities for achieving better quality of the denoised image.

Values of threshold in a series of possibilities $\delta(\mathbf{n})$ are defined and related to each element **n** of this series.

To consider the reconstructed image quality the normalized cross correlation (NCC) between the original and the denoised images is estimated.

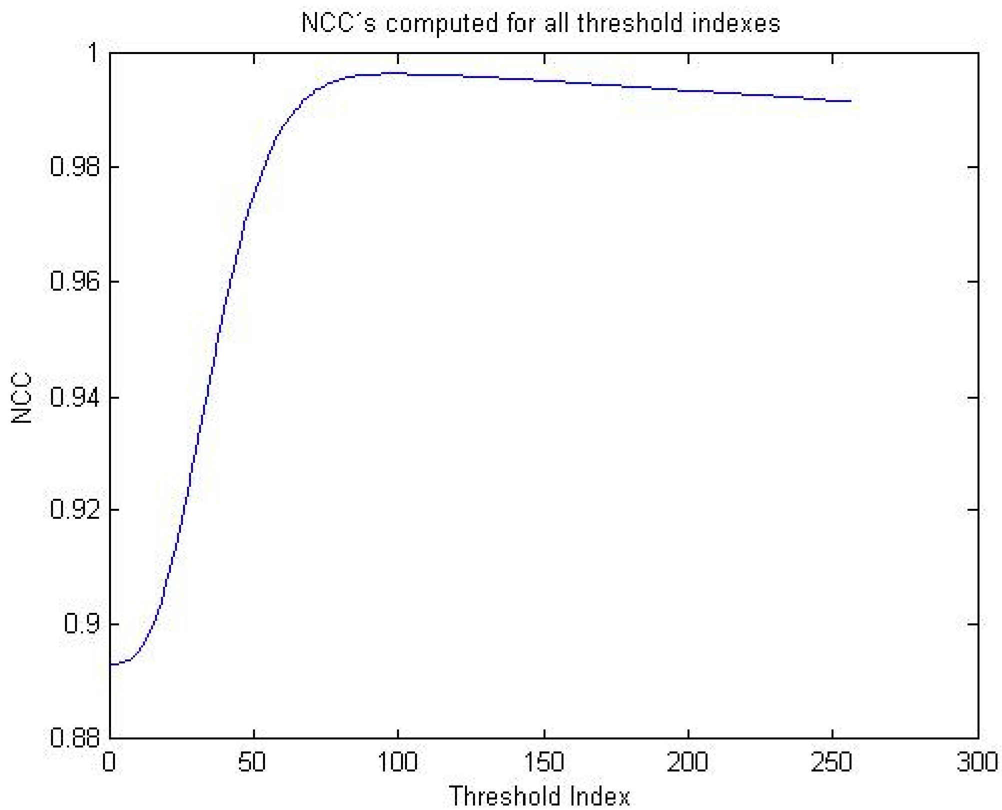
In this case when more correlated are the images better is the δ.

Then the best threshold value for a given image is found automatically

- by the system considering best quality possible for the restored image when all others parameters are defined.

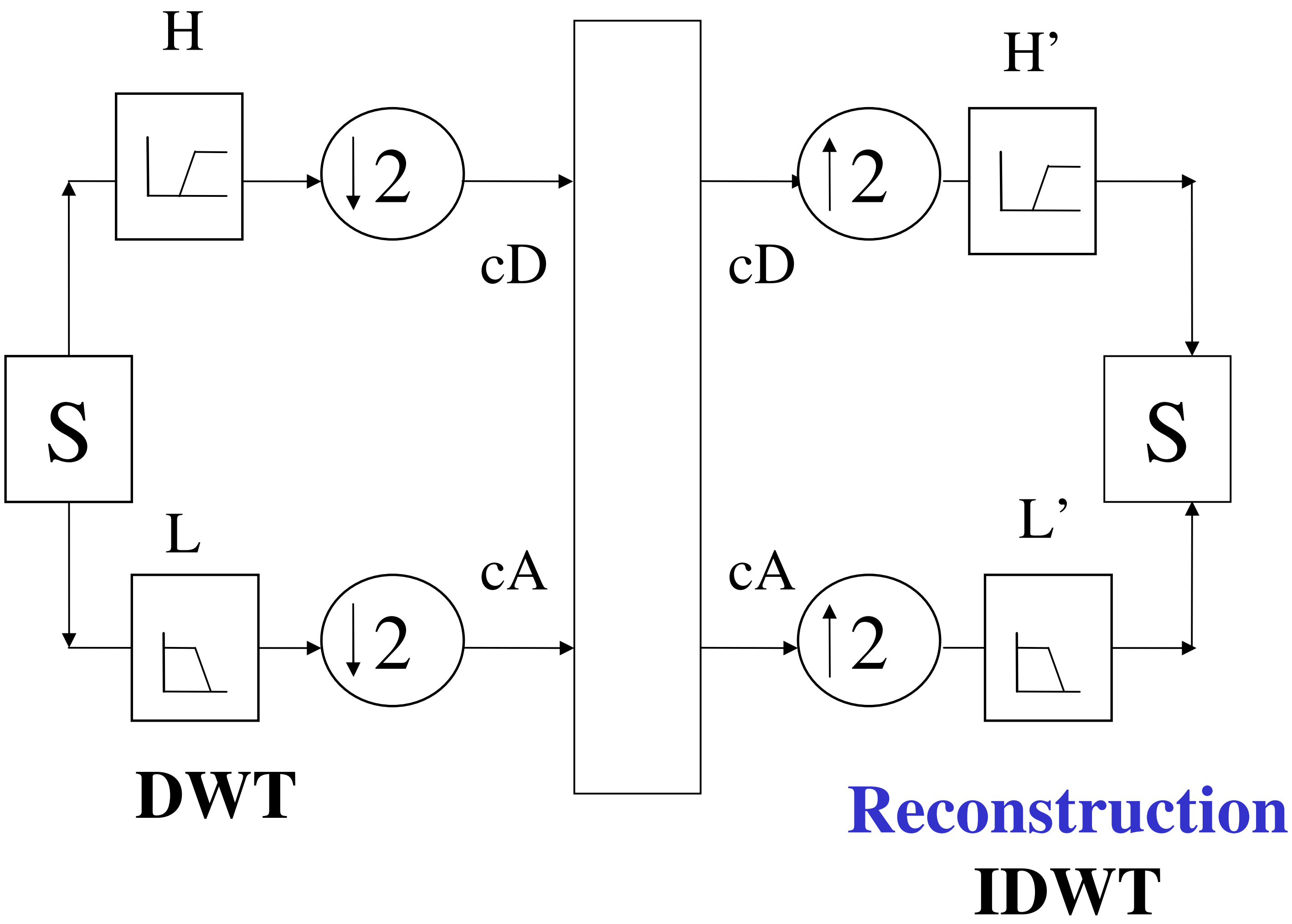
- Such search is put in an admissible computational time by using discrete possibilities previous delimited the best δ is found by a function of complexity O (log(**n**) .

Optimal reconstruction threshold



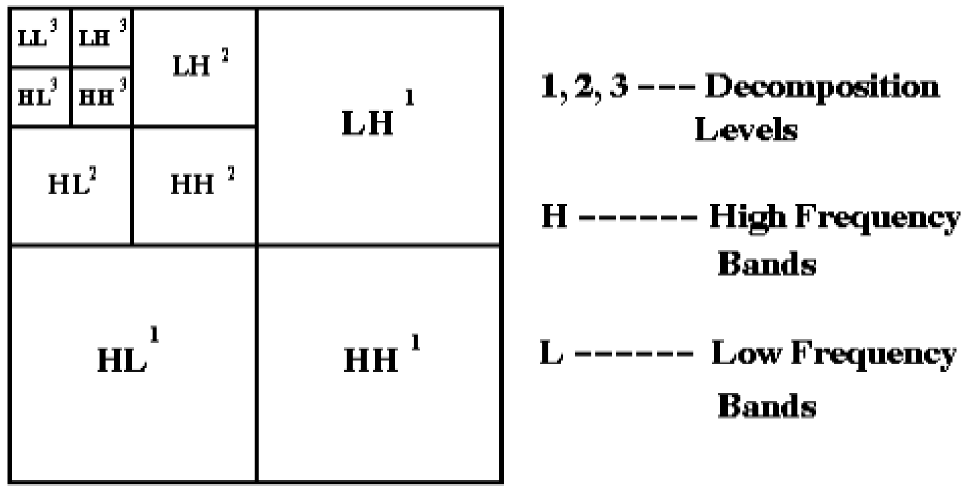
Example of the concave function relating threshold index and NCC for the best result of the base Biorthogonal 1.3.

Low and High pass filters

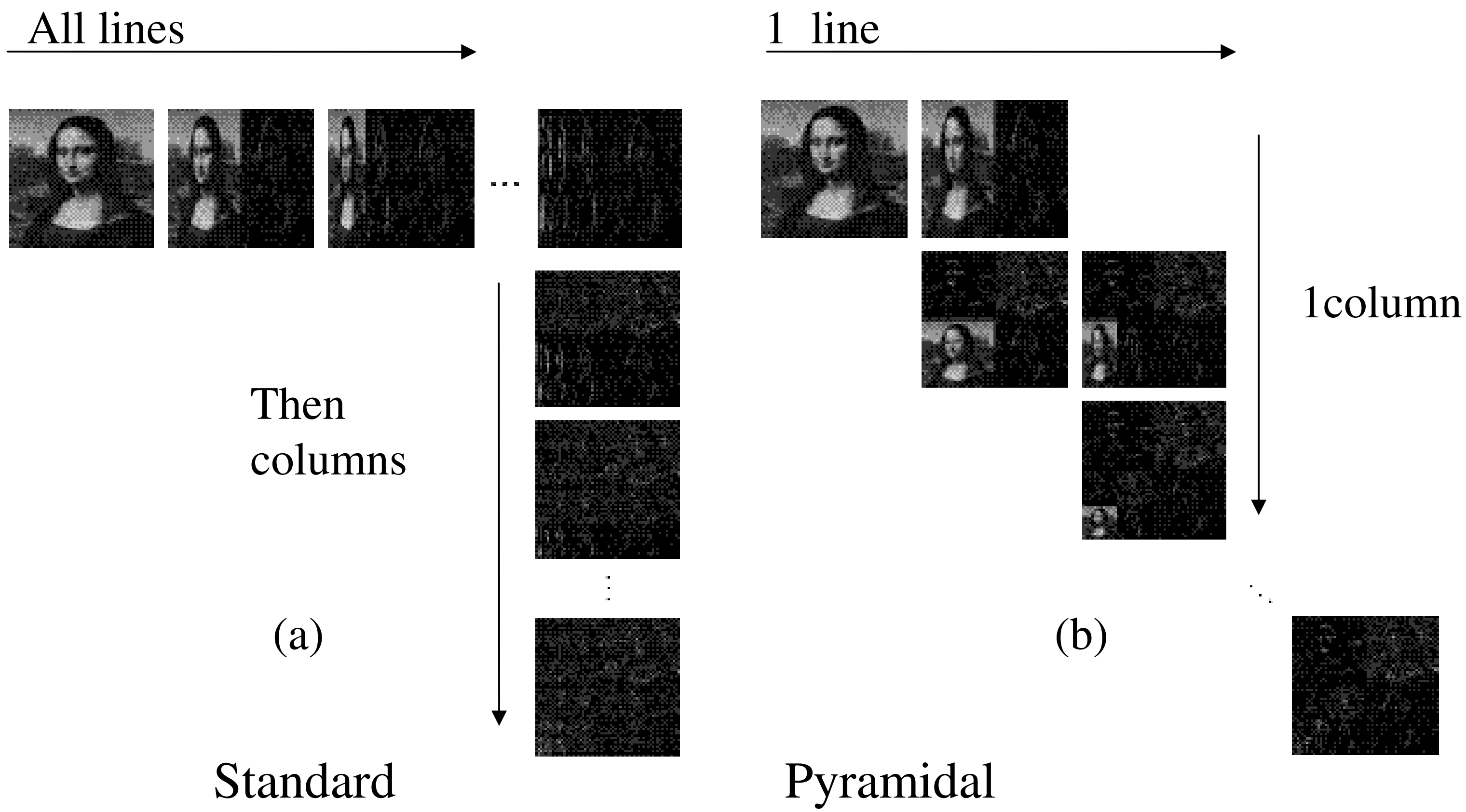


Decomposition on levels 3

- (**j=**3) levels of high (**H**) and low (**L**) sub bands.



2d


All lines
Then
columns
(a)
Standard
1 line
1column
(b)
Pyramidal

Steps used on experiments with synthetic added noise images

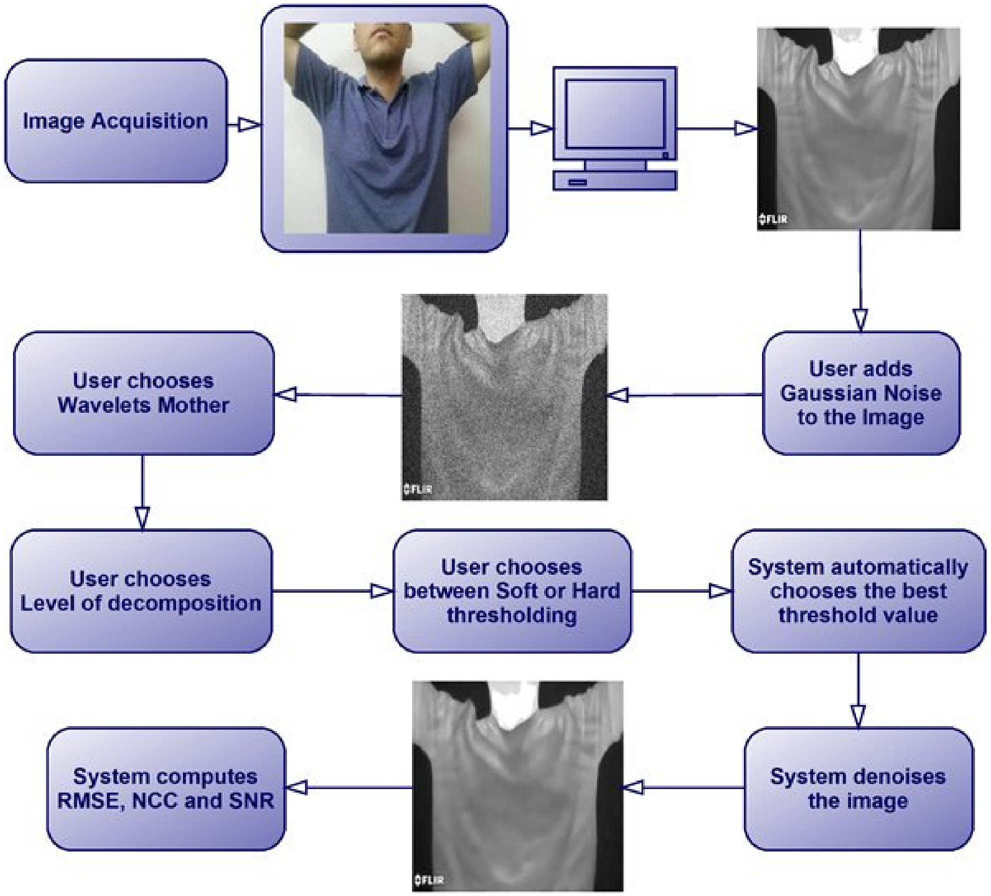
Step 1: Image acquisition and storage as a raw data.

Step 2: Gaussian noise addition. Three levels of a standard deviation value (σ_{noise} = 5, 25 and 50) are added.

Step 3: Define the type of wavelet, level of adaptive decomposition and the threshold process. Then the system select the coefficient for threshold based on the normalized cross correlation (NCC) that produces greater correlation

Step 4: Image restoration

Step 5: Verification



Results

- For the 8 images, each of the 108 bases are tested for levels 3 and 4 of the decomposition (L3 and L4), and the 2 possible way of coefficient thresholding (soft and hard).

- Each configuration has been considered for the images with added Gaussian noise at three different levels, with the best thresholding value automatically computed, resulting in a total of **10368 experiments**

MATLAB 7.12.0 (R2011a)

For each configuration the evaluators are:

$$RMSE = \sqrt{\left[\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[G(x,y) - F(x,y) \right]^2 \right]} \qquad \textbf{(1)}$$

$$SNR_{ms} = \frac{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} G(x,y)^2}{\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[G(x,y) - F(x,y) \right]^2} \qquad \textbf{(2)}$$

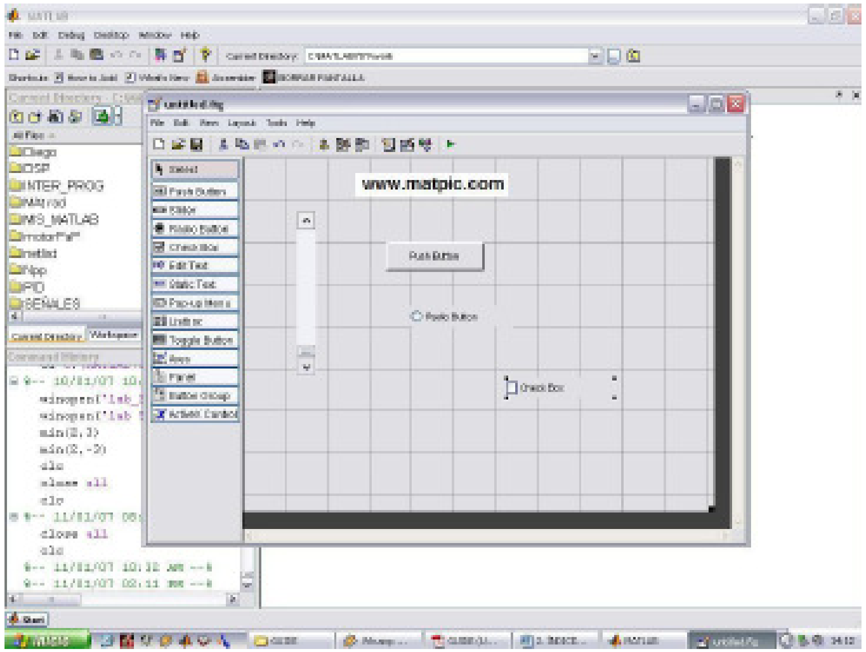
$$SNR_{rms} = \sqrt{SNR_{ms}} \qquad \textbf{(3)}$$

$$PSNR = 20 \log_{10} \left(\frac{2^n - 1}{RMSE} \right) \qquad \textbf{(4)}$$

Used WT types

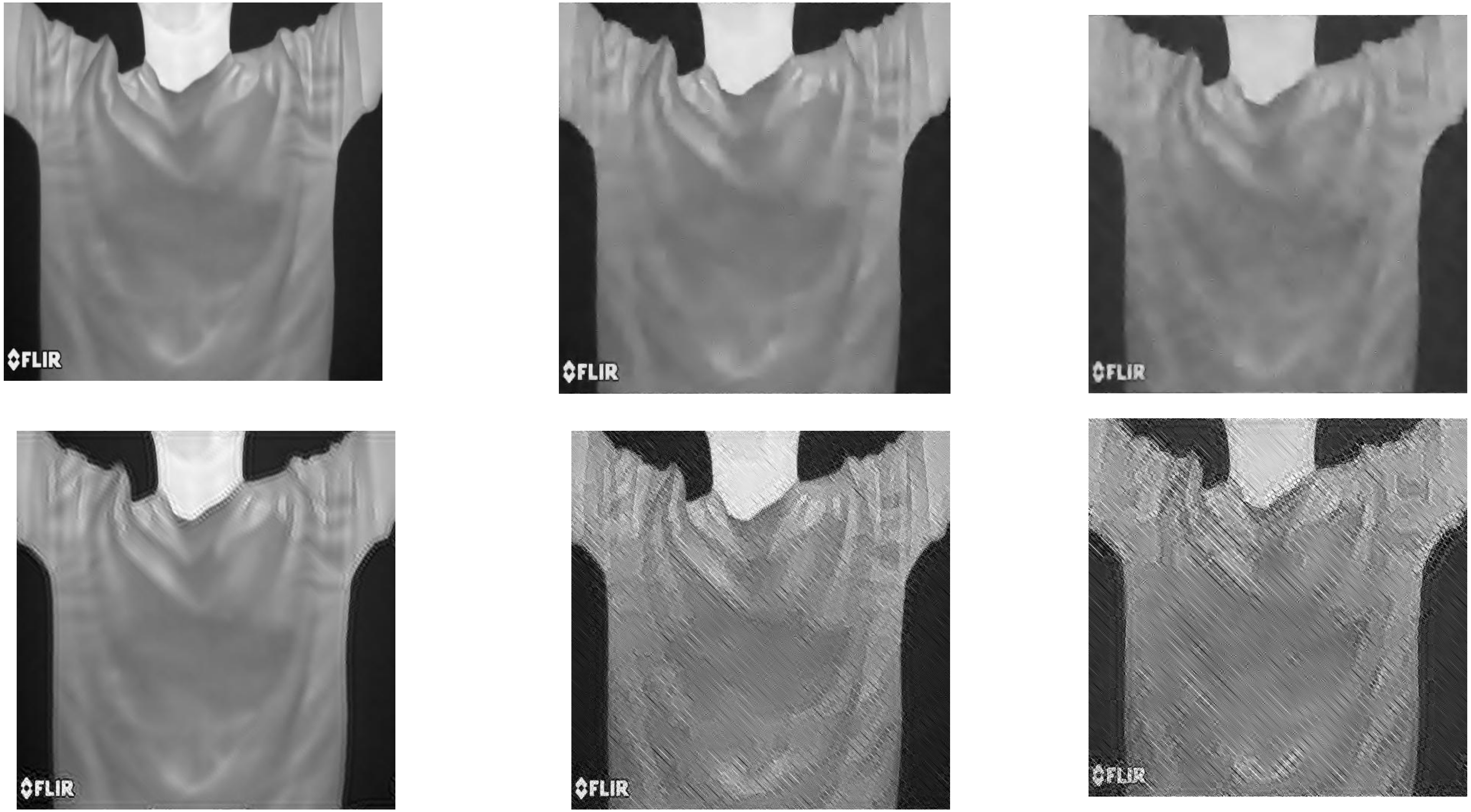
	A	B	C	D	E	F	G	H	I	J	K
1	Bior 1.1	Bior 3.7	db 1	db 11	db 21	db 31	db 41	rbio 2.2	rbio 5.5	sym 10	sym 20
2	Bior 1.3	Bior 3.9	db 2	db 12	db 22	db 32	db 42	rbio 2.4	rbio 6.8	sym 11	sym 21
3	Bior 1.5	Bior 4.4	db 3	db 13	db 23	db 33	db 43	rbio 2.6	sym 2	sym 12	sym 22
4	Bior 2.2	Bior 5.5	db 4	db 14	db 24	db 34	db 44	rbio 2.8	sym 3	sym 13	sym 23
5	Bior 2.4	Bior 6.8	db 5	db 15	db 25	db 35	db 45	rbio 3.1	sym 4	sym 14	sym 24
6	Bior 2.6	coif 1	db 6	db 16	db 26	db 36	Dmey	rbio 3.3	sym 5	sym 15	sym 25
7	Bior 2.8	coif 2	db 7	db 17	db 27	db 37	Haar	rbio 3.5	sym 6	sym 16	sym 26
8	Bior 3.1	coif 3	db 8	db 18	db 28	db 38	rbio 1.1	rbio 3.7	sym 7	sym 17	sym 27
9	Bior 3.3	coif 4	db 9	db 19	db 29	db 39	rbio 1.3	rbio 3.9	sym 8	sym 18	sym 28
10	Bior 3.5	coif 5	db 10	db 20	db 30	db 40	rbio 1.5	rbio 4.4	sym 9	sym 19	-

- http://bit.ly/TeUR1C

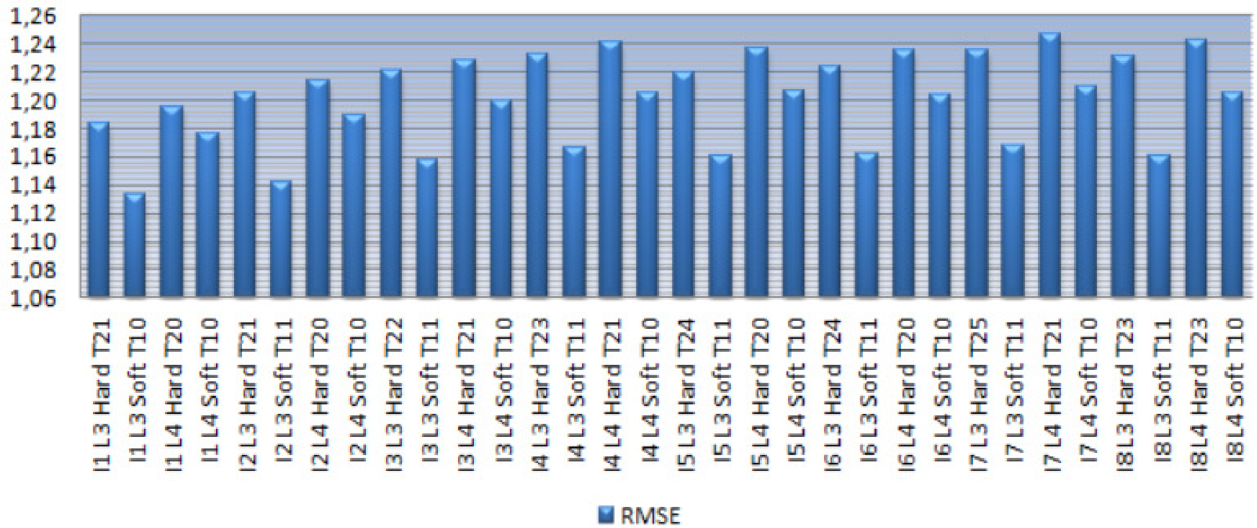


All image used are acquired by a Flir S45 camera (with sensibility of 0.08ºC) in 640x480 resolution and encoded using 8 bit per pixels.

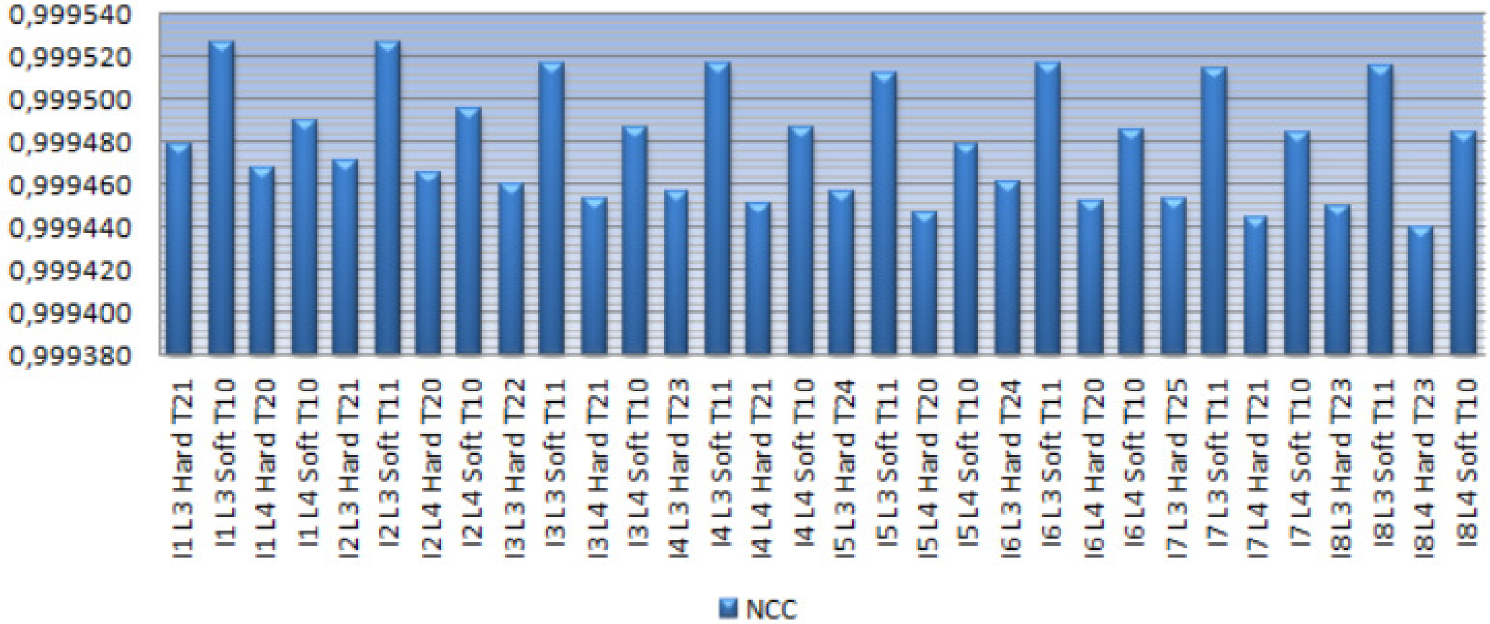
Restoration by best and worst results



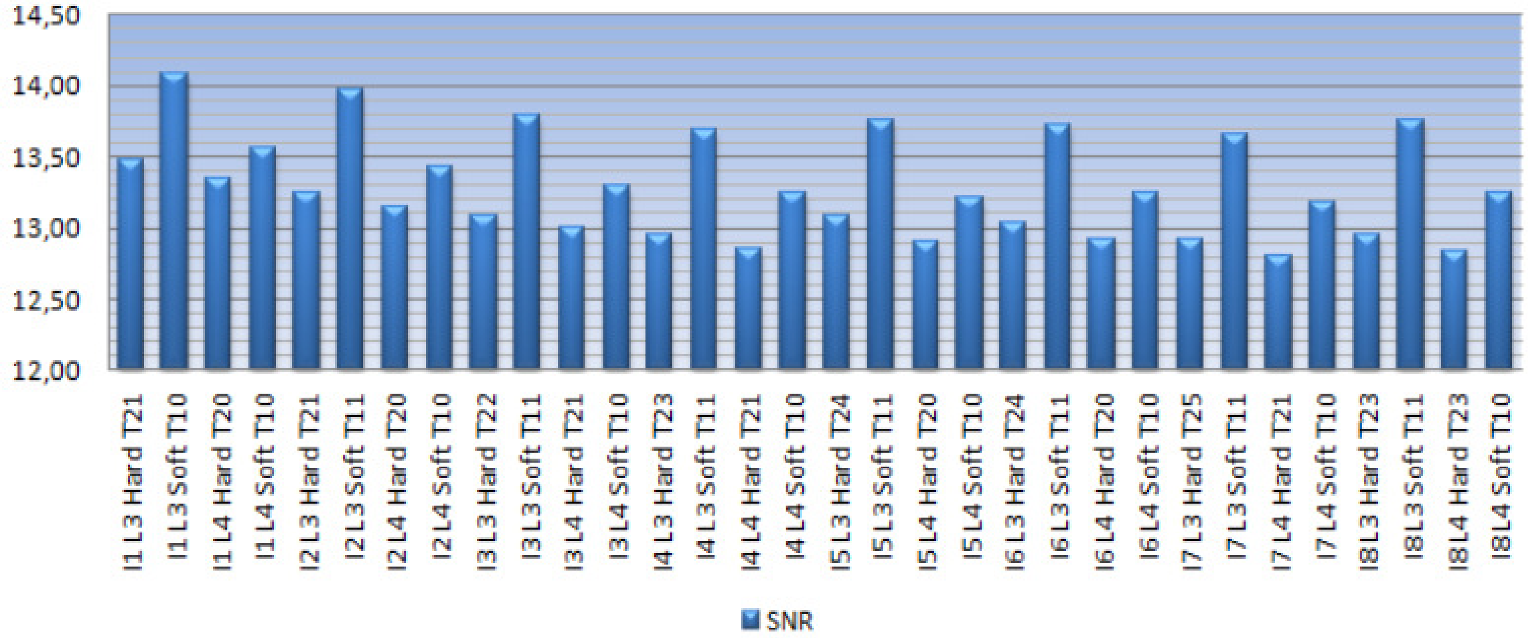
RMSE - Low noise



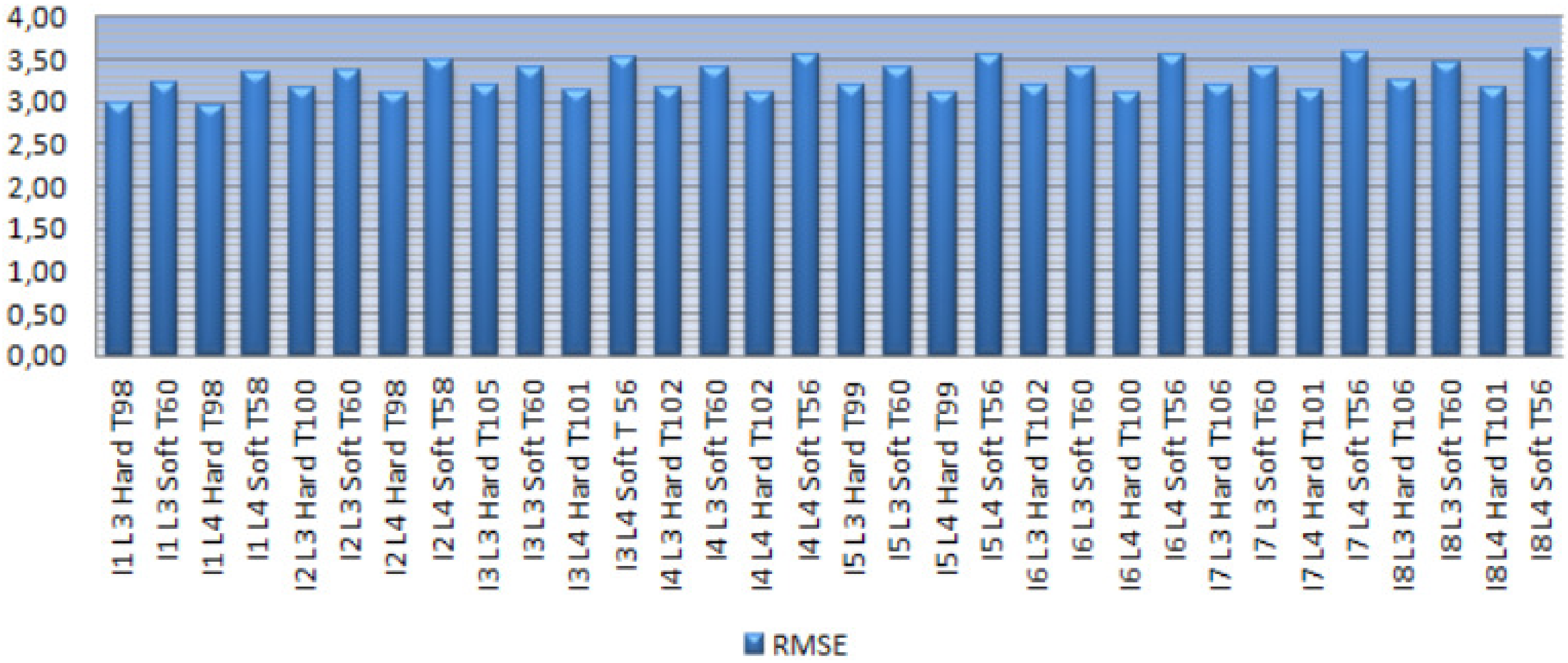
NCC low noise



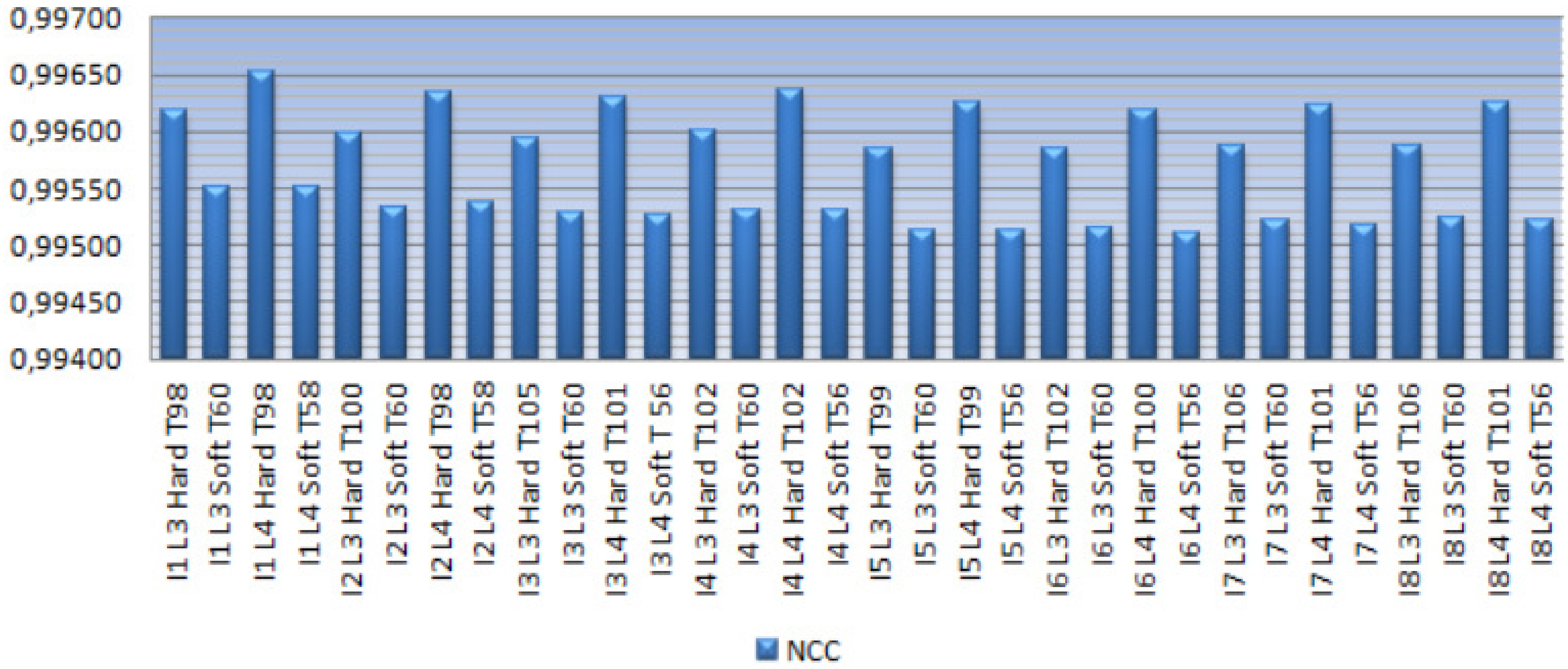
SNR – low noise



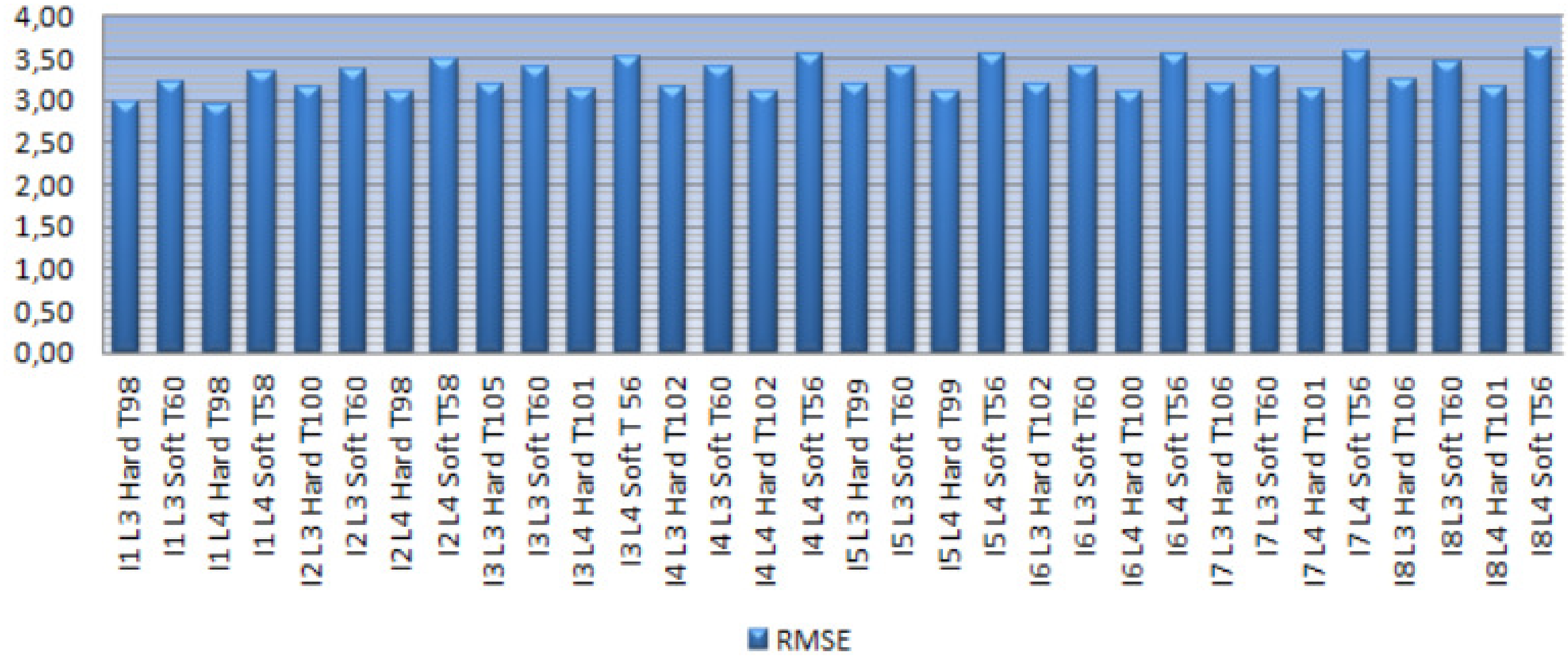
RMSE - medium noise



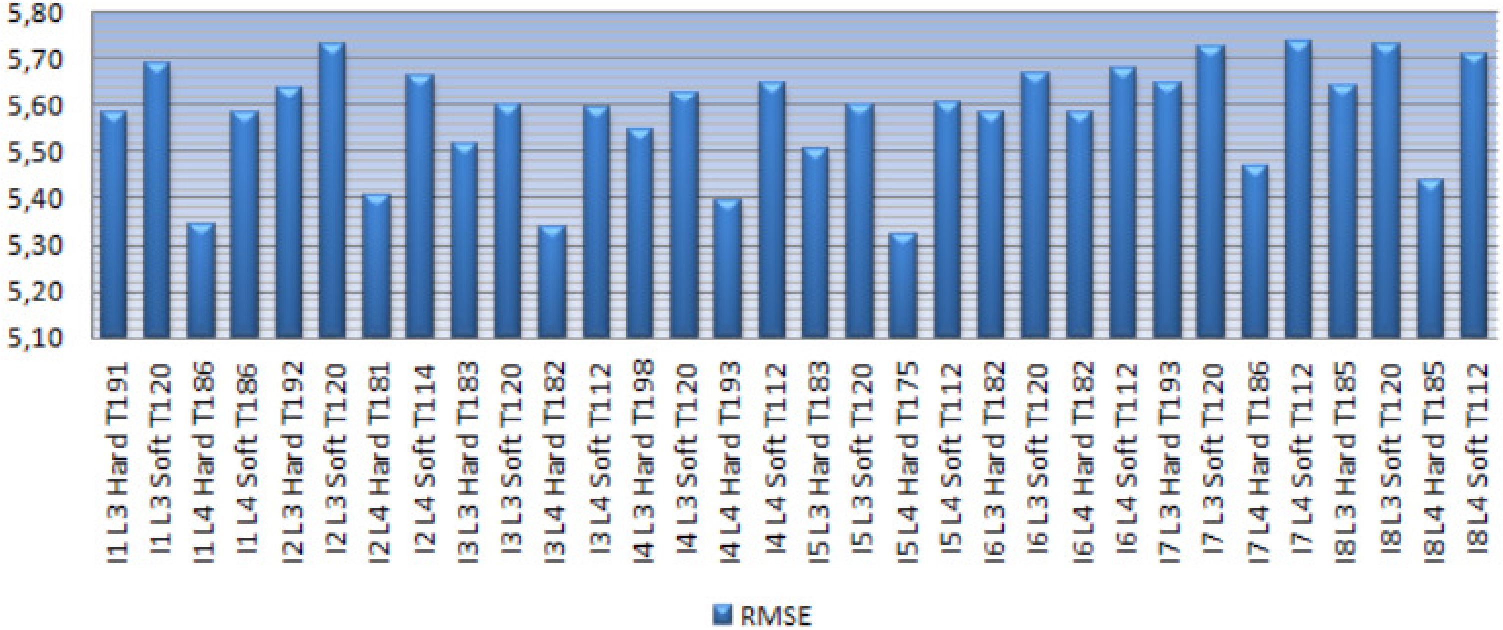
NCC medium noise



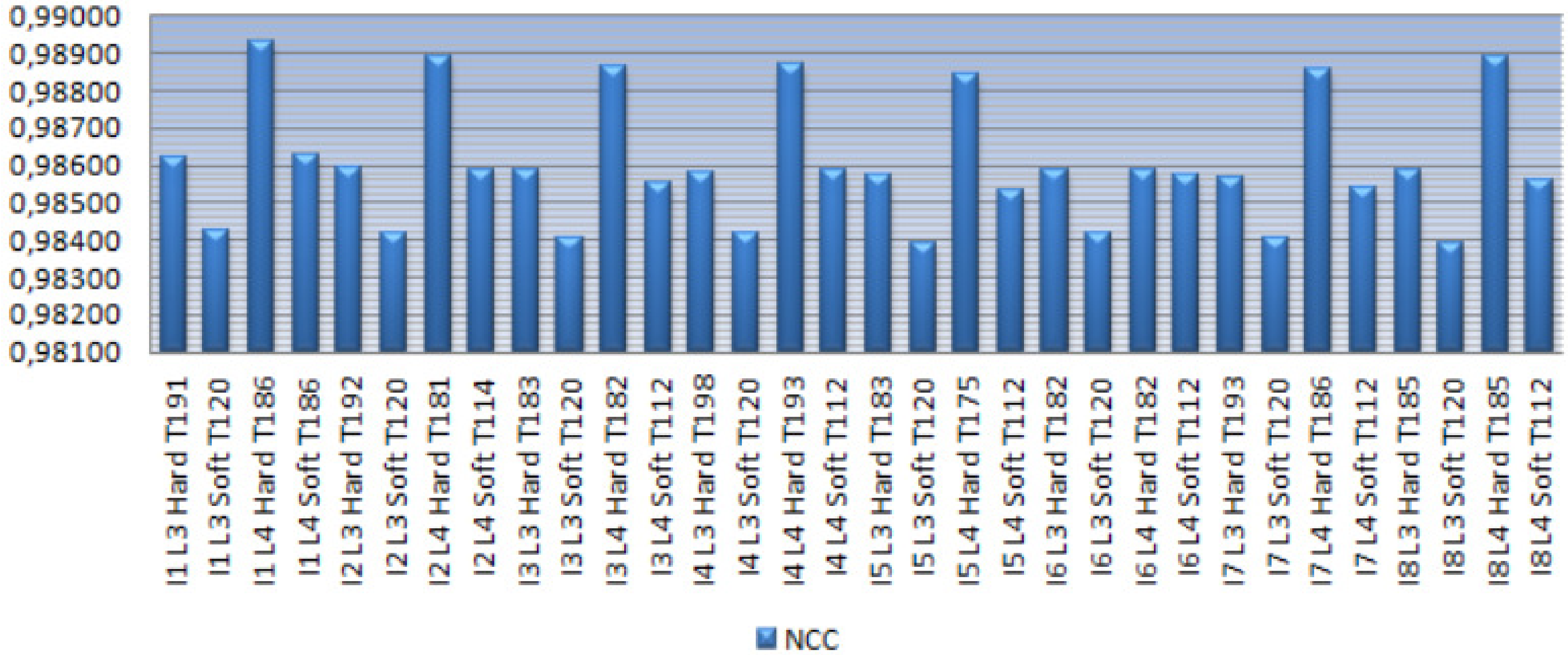
PSNR - medium noise level



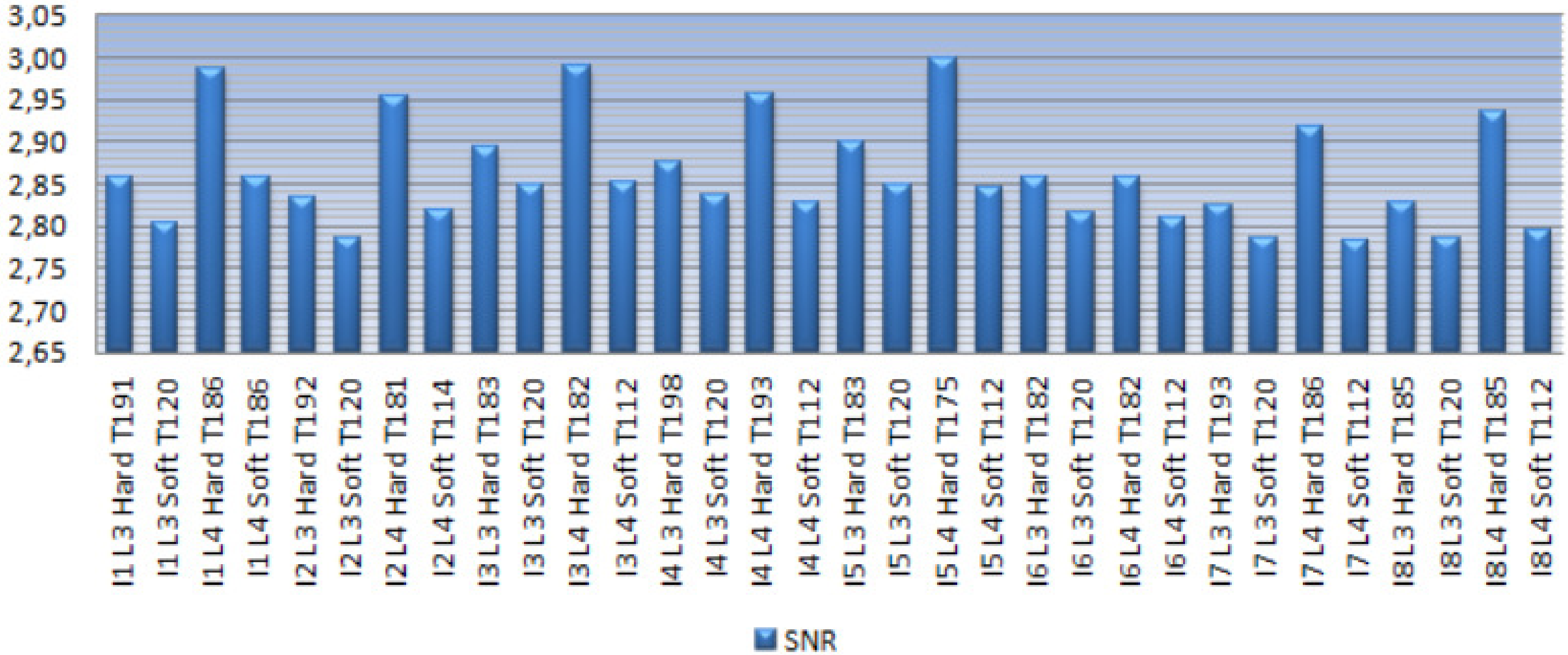
RMSE - high noise level



NCC - high noise level



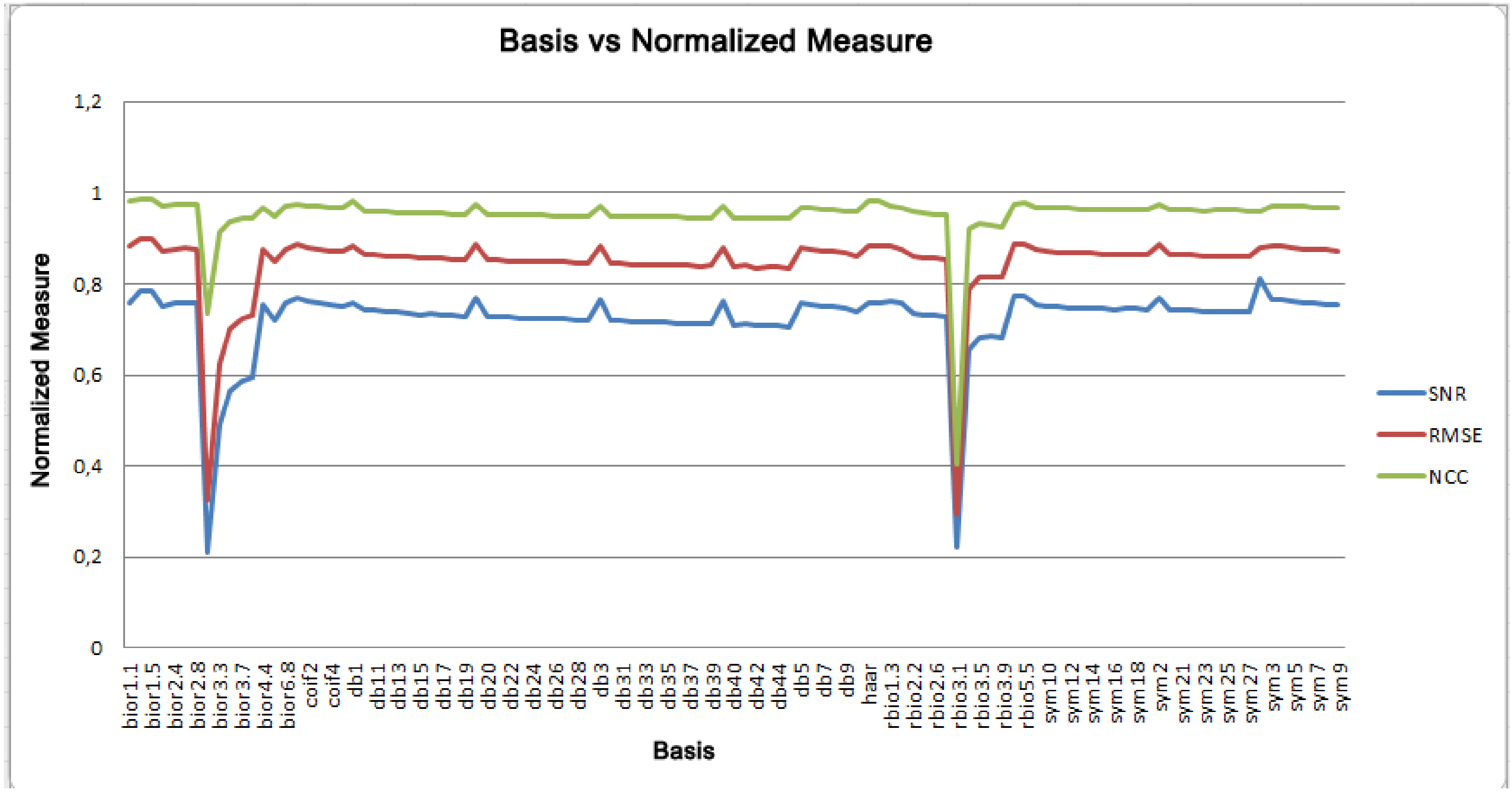
PSN - high noise level



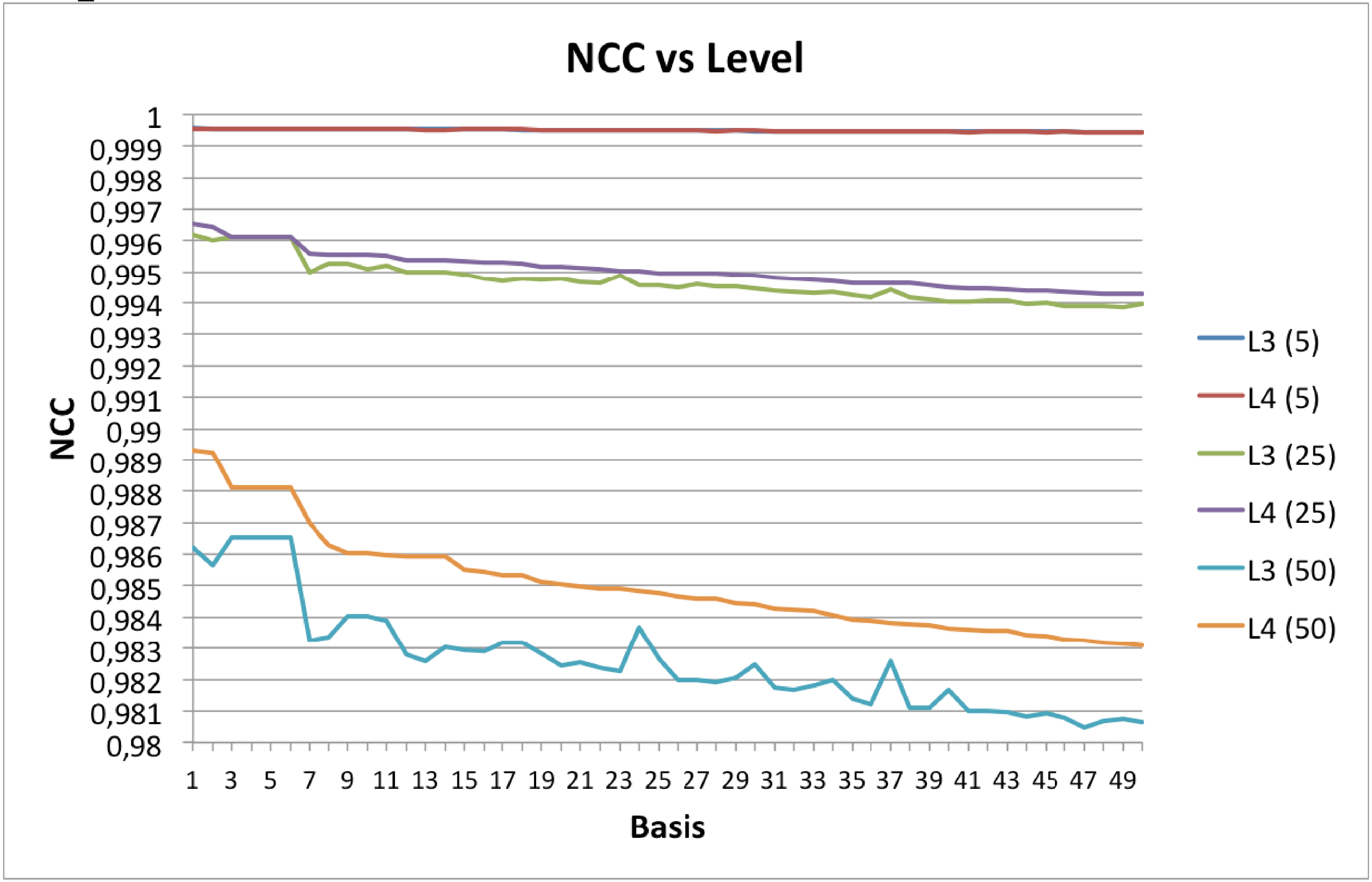
Comparison of the average NCC values for all images on all noise level for the used denoising methods.

	A	B	C	D	E	F	G	H	I	J	K
1	0.9933	0.9896	0.9933	0.9906	0.9897	0.9891	0.9887	0.9908	0.9927	0.9915	0.9911
2	0.9938	0.9897	0.9925	0.9905	0.9896	0.9891	0.9886	0.9905	0.9917	0.9914	0.9911
3	0.9937	0.9917	0.9922	0.9903	0.9895	0.9890	0.9886	0.9901	0.9925	0.9914	0.9910
4	0.9921	0.9894	0.9919	0.9902	0.9895	0.9890	0.9886	0.9898	0.9922	0.9913	0.9909
5	0.9924	0.9918	0.9916	0.9901	0.9894	0.9889	0.9885	0.9308	0.9922	0.9913	0.9910
6	0.9924	0.9925	0.9913	0.9901	0.9894	0.9889	0.9909	0.9863	0.9920	0.9912	0.9911
7	0.9923	0.9921	0.9912	0.9900	0.9893	0.9888	0.9933	0.9877	0.9919	0.9912	0.9910
8	0.9705	0.9919	0.9910	0.9899	0.9893	0.9888	0.9933	0.9874	0.9917	0.9912	0.9908
9	0.9868	0.9917	0.9908	0.9898	0.9892	0.9888	0.9920	0.9871	0.9917	0.9912	0.9935
10	0.9890	0.9915	0.9907	0.9897	0.9892	0.9887	0.9916	0.9925	0.9916	0.9910	-

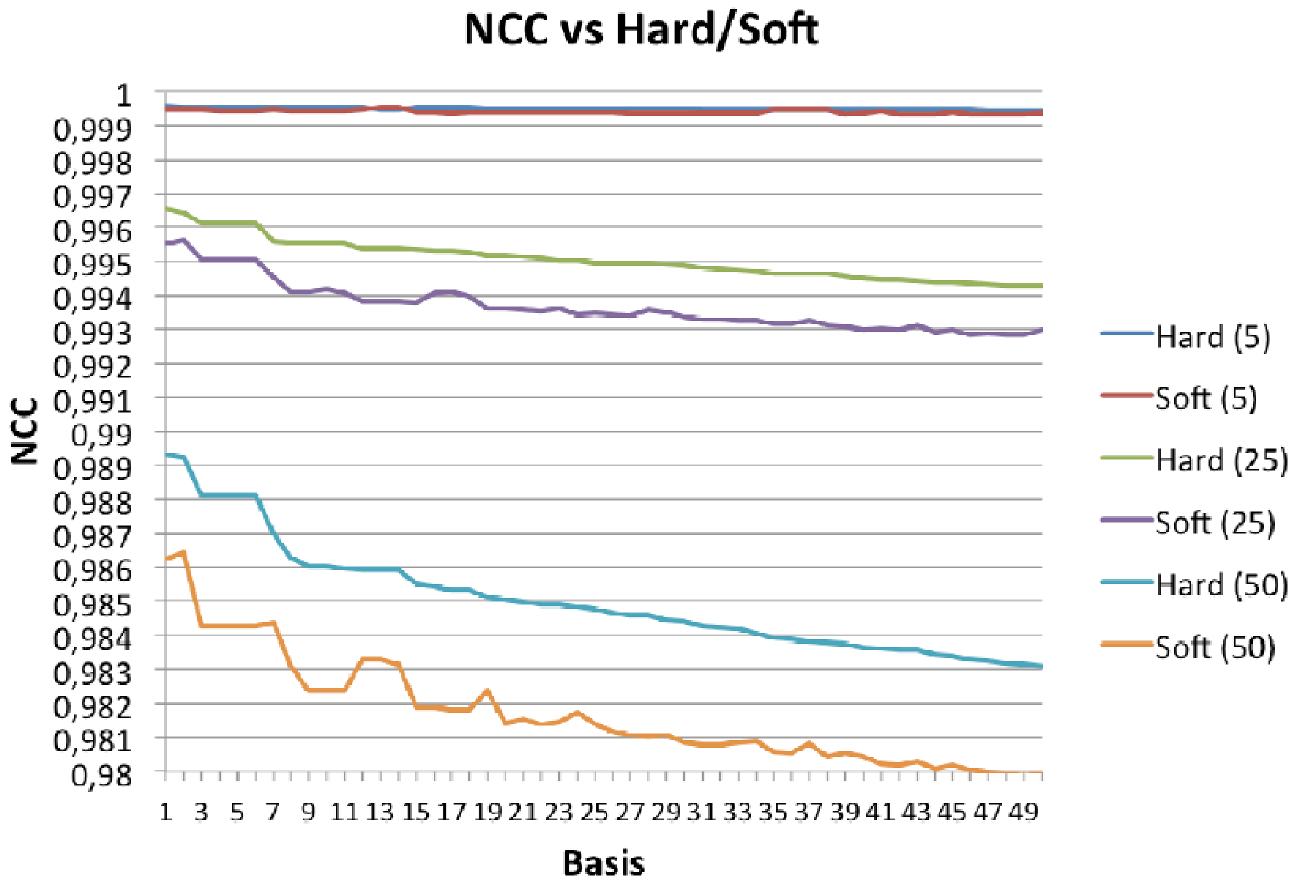
Comparing the measures SNR, NCC and RMSE for each type of wavelet used.



top 50 results



top 50 results



The 10 best combinations of characteristics for low noise level

Base	Level	H/S	NCC	SNR	RMSE
Coif 1	L3	H	0.999557	117.865253	1.133402
Coif 1	L4	H	0.999552	117.273865	1.146983
Sym 2	L3	H	0.999551	117.129401	1.135497
Db 2	L3	H	0.999551	117.129401	1.135497
Sym 3	L3	H	0.999548	116.427425	1.14867
Db 3	L3	H	0.999548	116.427425	1.14867
Sym 2	L4	H	0.999547	116.664894	1.148594
Db 2	L4	H	0.999547	116.664894	1.148594
Bior 2.6	L4	H	0.999547	116.628034	1.142708
Rbio 5.5	L4	H	0.999547	116.483195	1.143878

Results for the best case of each noise level.

Noise l.	Base	Dec.l.	H\S	Thr. index	SNR	RMSE	NCC
5	coif 1	3	h	18	117.865	1.098	0.999
25	bior 1.3	4	h	98	33.615	3.857	0.996
50	bior 1.3	4	h	183	16.850	7.611	0.989

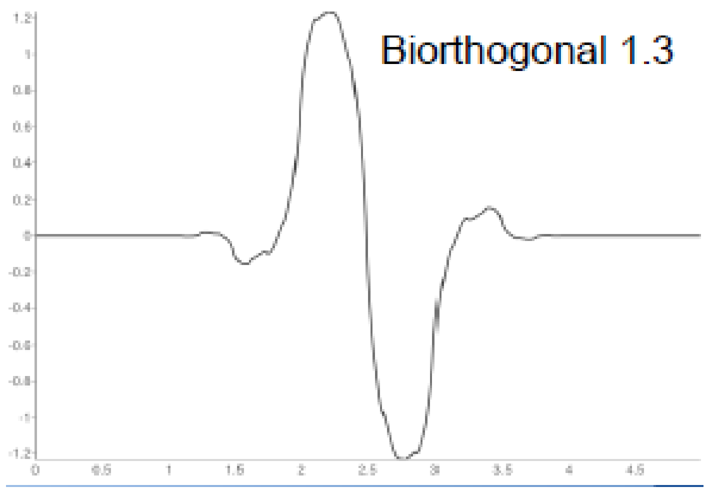
Conclusion :

Averaging all noise level:

The most relevant are:
Coiflet 1, Symmlet 2, Daubechie 2, Symmlet 3, Daubechie 3, Biortogonal 2.6 and Reverse biortogonal 5.5.

The hard threshold is always better.

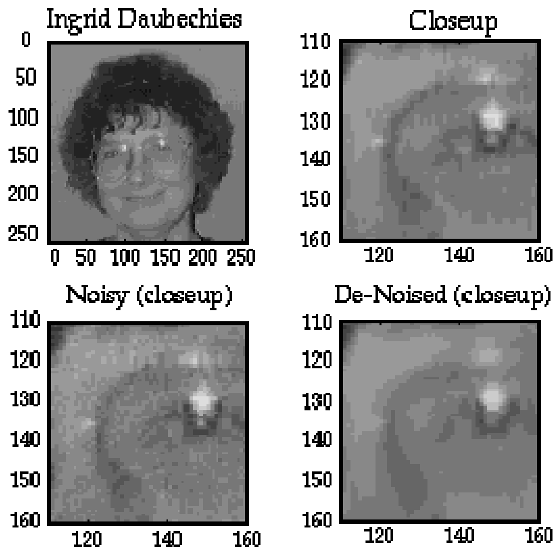
For low level of noise only the three levels of decomposition can be used.



Famous example of Daubechies (1993) denoise

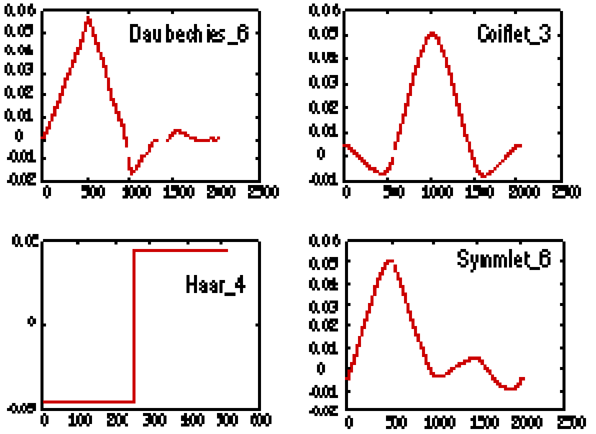
Donoho denoise:

- Coiflets-3
- threshold
- inverse



Second part

- Use these conclusions for the database project.



Steps to perform an efficient restoration scheme for infrared images considering the noise level.

1: Image acquisition and storage as a raw data

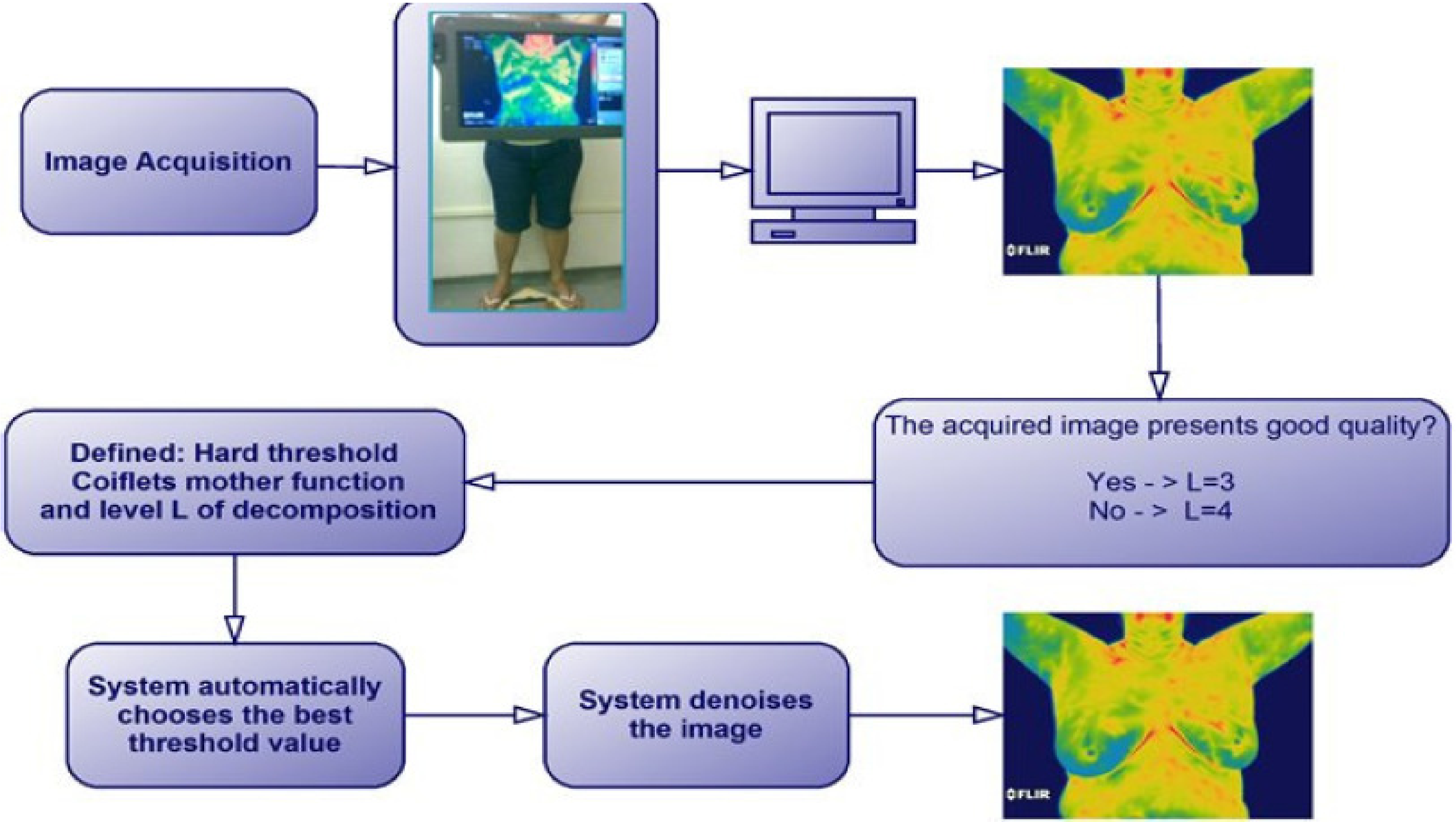
2: Evaluation of noise level and decision about decomposition in level 3 or 4.

3: Coiflet wavelet and hard threshold are used.

4: Coefficients for thresholding is select automatically based on the NCC.

5: The image is reconstructed using the modified coefficients.

Restoration of real infrared of whatever noise levels



Results

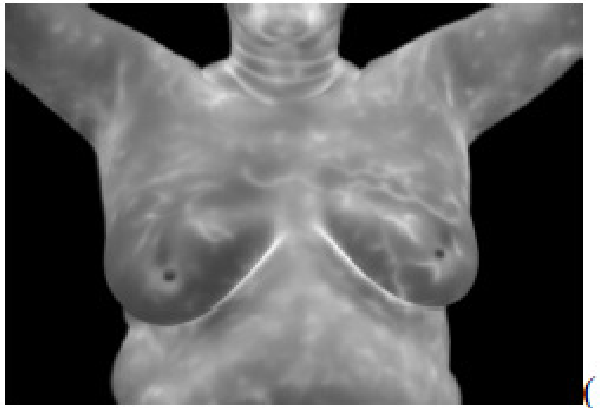
Original : 49.519 bytes (1),

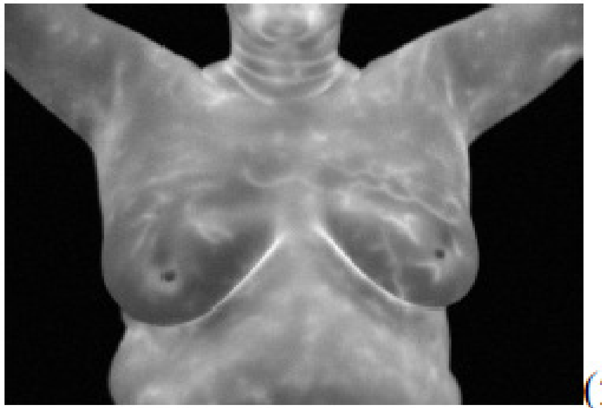
image after storage and transmission:
50.846 byte (2) and

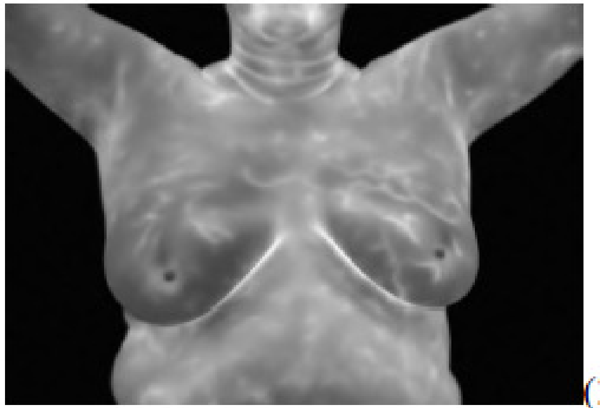
denoised image by the proposed scheme:
15.869 bytes (3).

Comparing achieved characteristics for typical breast image.

images	SNR	RMSE	NCC	Size (bytes)
1 - 2	5.9197	2.2273	0.8202	50.846
1 - 3	16.0751	0.8202	0.9997	15.869


(1)

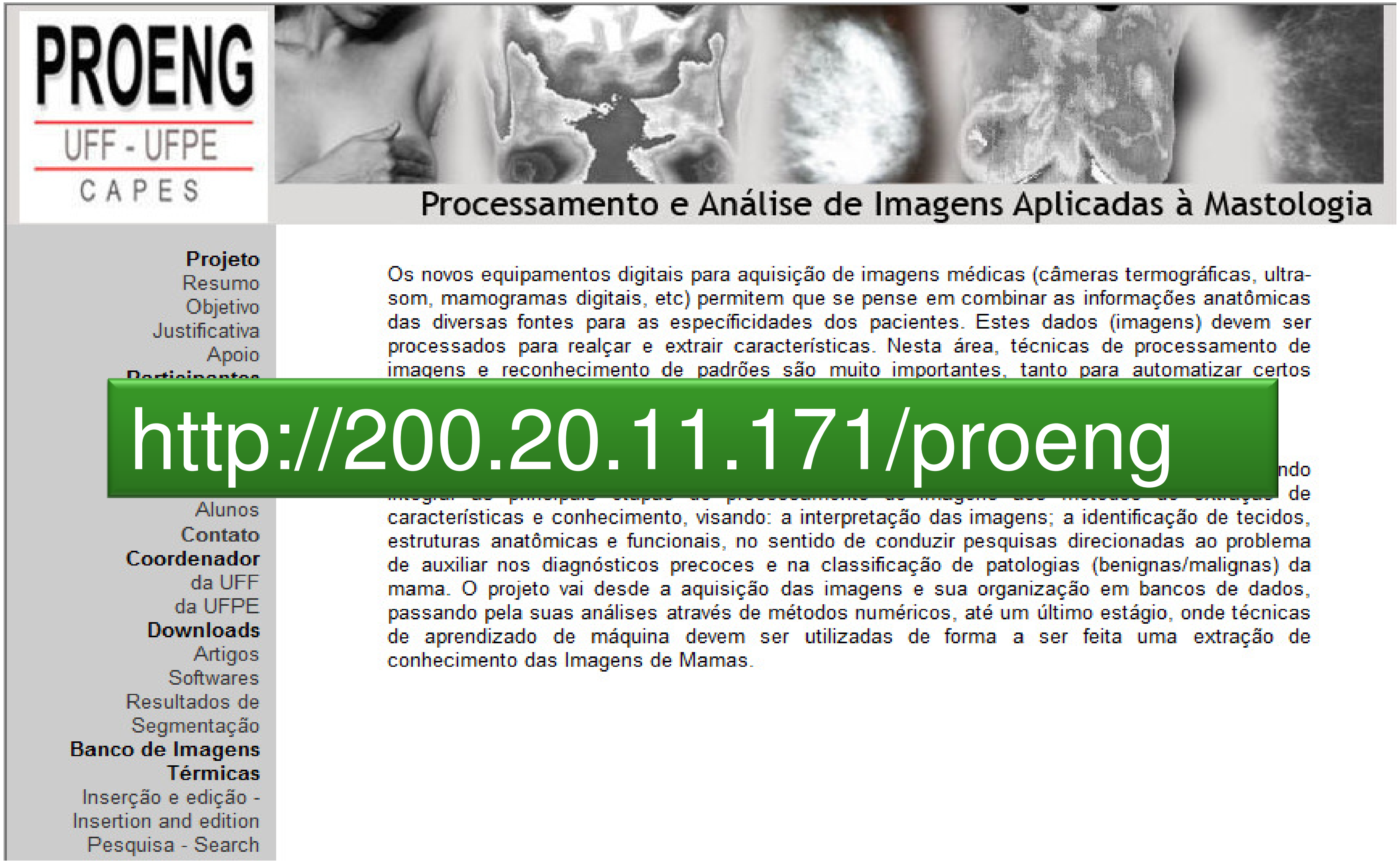

(2)


(3)

Thanks

- aconci@ic.uff.br
- www.ic.uff.br/~aconci
- http://200.20.11.171/proeng/

PROENG's WebSite



http://200.20.11.171/proeng

Transformada de *Wavelet* Contínua

Transformada de *Wavelet* Contínua é a integral ao longo do tempo de um sinal multiplicado por uma escala, e deslocado por uma função *wavelet* (*Psi*), também chamada *wavelet* mãe:

$$C(escala\ , \ posição\ ) = \int_{-\infty}^{\infty} f(t)\,\Psi(escala\ , \ posição\ , \ t)\, dt$$

O número ao lado do nome da wavelet representa o número de momentos nulos !

***a* define a escala e *b* a translação**

$$\Psi_{a\,,\,b}(t) = \frac{1}{\sqrt{a}}\,\Psi\left(\frac{t-b}{a}\right), \quad a \neq 0\,, \quad b \in \Re$$

Repare que mais de uma forma (função base ou mãe) pode ser usada para **gerar uma familia**

Para ser considerada uma wavelet, uma função inversivel tem que:

Ter uma área total NULA sob a curva da função (ou integral nula) ; e

$$\int \Psi\,(t)\,dt = 0$$

Ter energia (ou integral do quadrado da função) finita,

$$C_{\Psi} = 2\,\pi \int |u|^{-1} \left|\hat{\Psi}(u)\right|^{2} du < \infty$$

Para que um *f* seja uma *Psi* Ψ

- Área zero

$$\int_{-\infty}^{\infty} \psi(t)dt = 0$$

energia finita:

$$\int_{-\infty}^{\infty} |\psi(t)|^2 dt$$

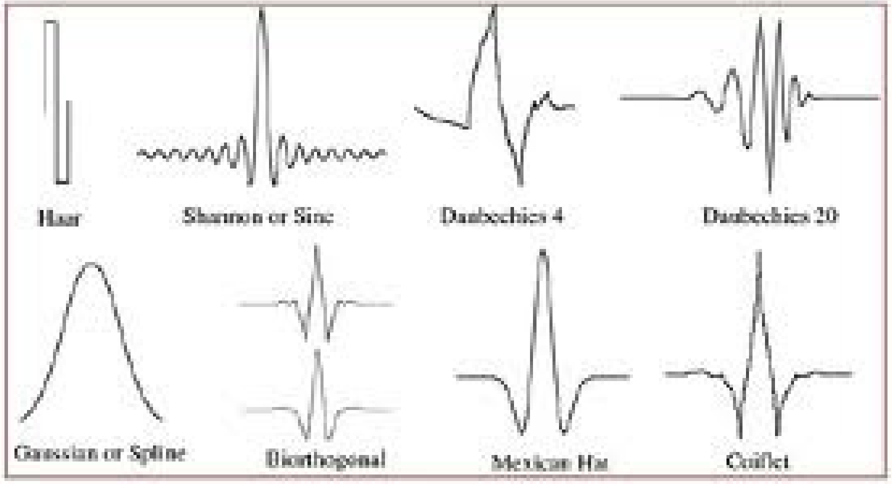


Figure 8

Examples of types of wavelets

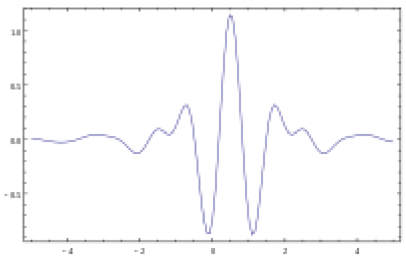
Tem que ter um
suporte compacto
- >
o que significa que ela deve
desaparecer fora de um
intervalo finito

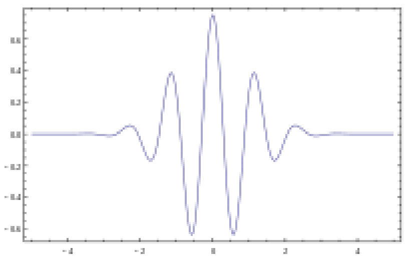
A Transformada de *Wavelets* contínua em *F(a,b)* é:
(repare que é uma função de dois parâmetros reais, *a* e *b*)

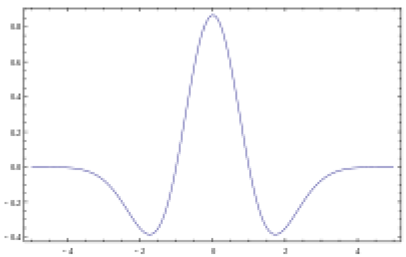
$$F(a,b) = \int f(t)\, \Psi_{a,b}\,(t)\, dt$$

A função $\Psi_{a,b}(t)$ é denominada função *wavelet e definida como*:

$$\Psi_{a\,,\,b}(t) = \frac{1}{\sqrt{a}}\,\Psi\left(\frac{t-b}{a}\right), \quad a \neq 0\,, \quad b \in \Re$$





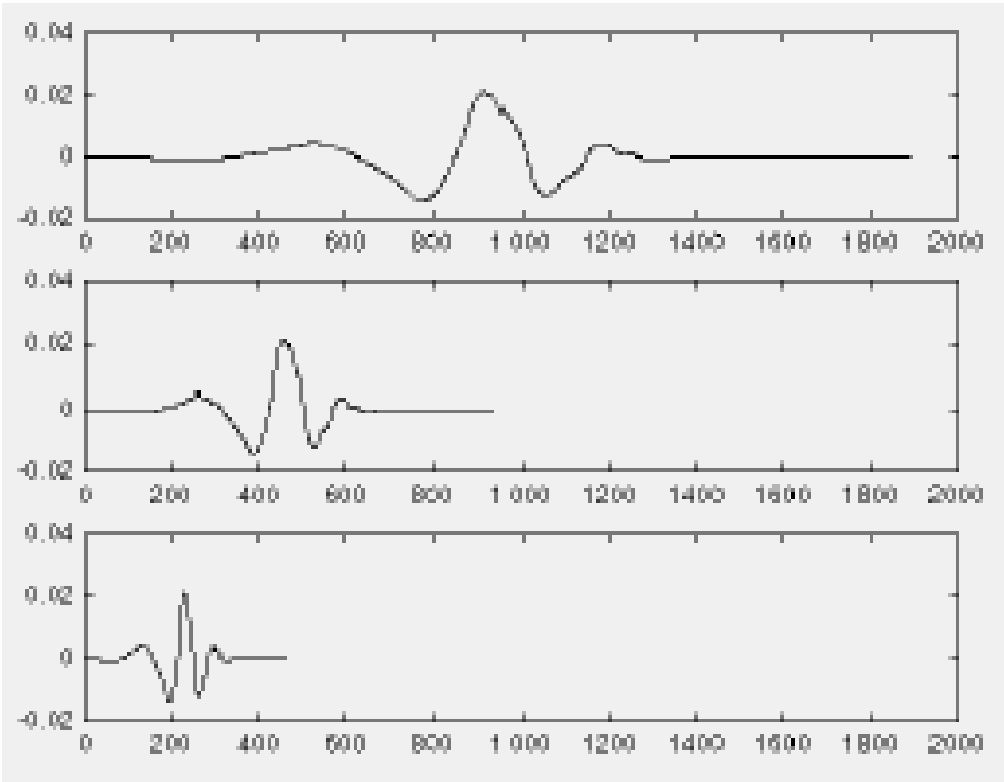


Meyer **Morlet** **Chapéu Mexicano**

A transformada de wavelet decompõe uma função definida no **domínio do tempo** em outra função, definida no **domínio do tempo e no domínio da frequência.**

Wavelet Transform

a - > scala



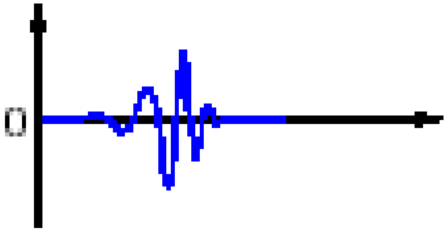
f(t)= Ψ*(t); a=1*

f(t)= Ψ *(2t); a=1/2*

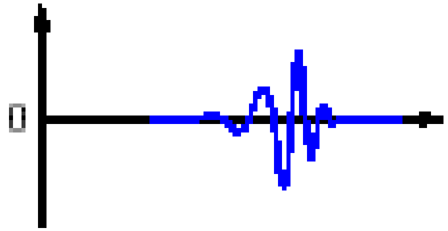
f(t)= Ψ *(4t); a=1/4*

Wavelet Transform

b - > Position



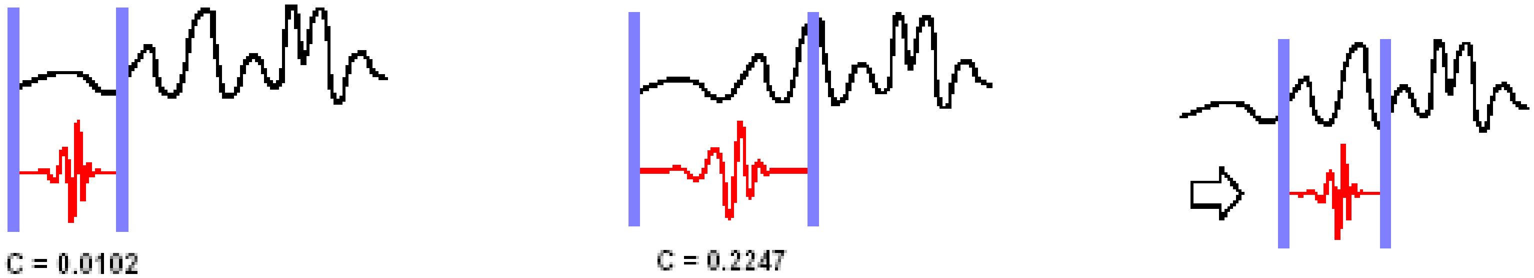
Wavelet



Same Function:
new location

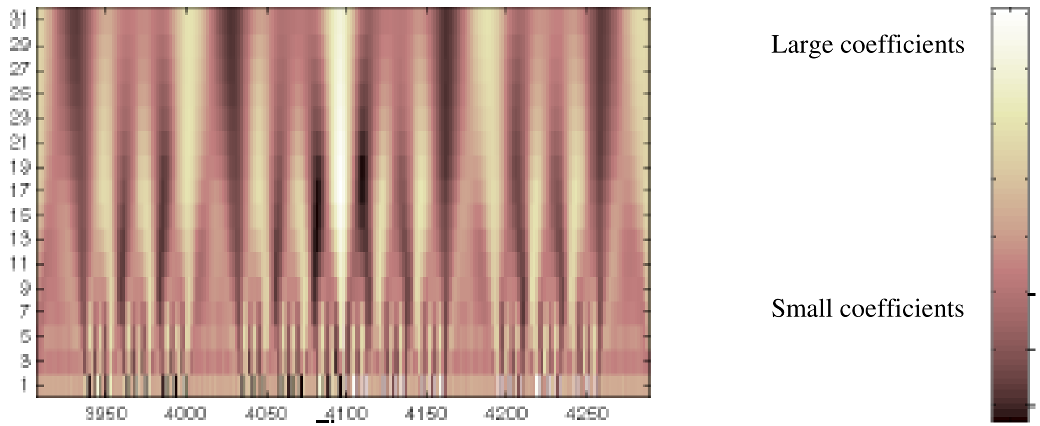
Wavelet Transform

scale, position & time:



mother wavelet ***a=1,b=0***

scale



time

Wavelets

$$F(a,b) = \int f(t)\, \Psi_{a,b}\ (t)\, dt$$

$$\Psi_{a,b}(t) = \frac{1}{\sqrt{a}}\, \Psi\left(\frac{t-b}{a}\right), \quad a \neq 0\,, \quad b \in \Re$$

$$C_{\Psi} = 2\,\pi \int |u|^{-1} \left|\hat{\Psi}(u)\right|^{2} du < \infty$$

$$\hat{\Psi}(0) = 0$$

A Transformada de *Wavelets* contínua em *F(a,b)* é:

$$F(a,b) = \int f(t)\, \Psi_{a,b}\,(t)\, dt$$

A função $\Psi_{a,b}(t)$ é denominada *wavelet*:

$$\Psi_{a,b}(t) = \frac{1}{\sqrt{a}}\Psi\left(\frac{t-b}{a}\right), \quad a \neq 0, \quad b \in \Re$$

As funções *wavelets* devem ter área zero e energia finita:

$$C_{\Psi} = 2\pi \int |u|^{-1} \left|\hat{\Psi}(u)\right|^2 du < \infty$$

condição de admissibilidade

Discrete WT

$$\Psi_{a,b}(t) = \frac{1}{\sqrt{a}} \Psi\left(\frac{t-b}{a}\right), \quad a = 2^j, b = k\ 2^j, \quad (j,k) \in Z^2$$

mother wavelet ***a=1,b=0 => j=0 e k=0***

Haar

- base Haar

$$\psi(x) \equiv \begin{cases} 1 & 0 \le x < \frac{1}{2} \\ -1 & \frac{1}{2} < x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$0 \le k \le 2^j - 1$$

$$\psi_{jk}(x) \equiv \psi\left(2^j x - k\right)$$

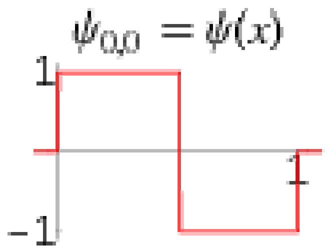
mother wavelet ***j=k=0***

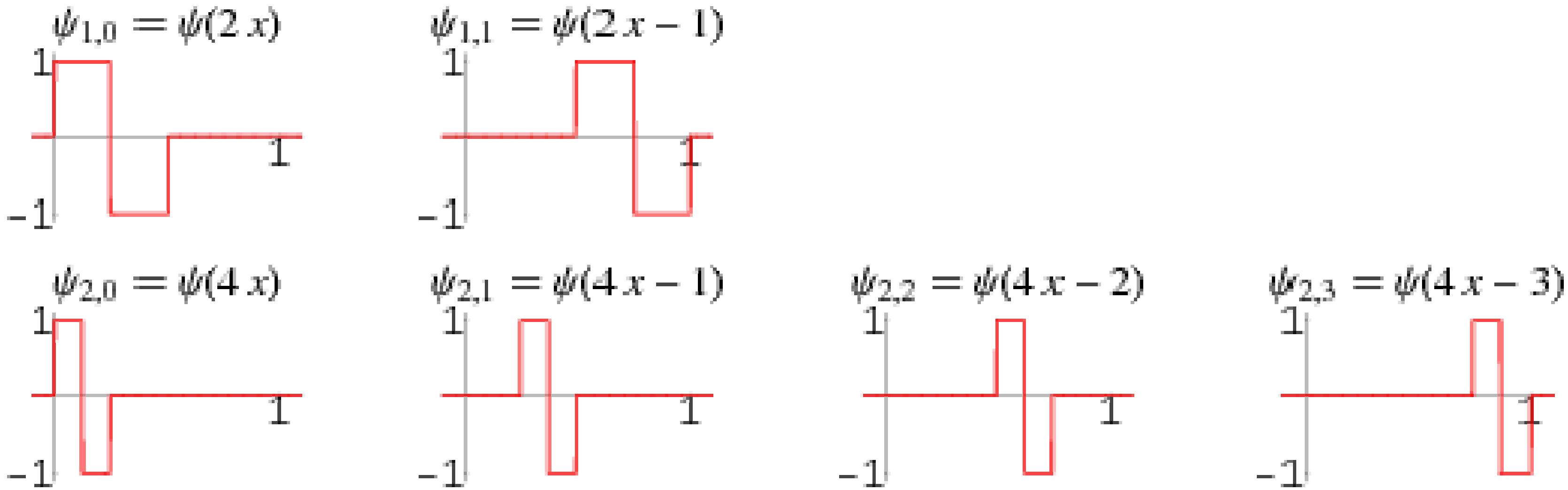
discrete Haar:

Alfred Haar – 1909

Particular case of Daubechies:

$$\psi(x) \equiv \begin{cases} 1 & 0 \le x < \frac{1}{2} \\ -1 & \frac{1}{2} < x \le 1 \\ 0 & \text{otherwise} \end{cases}$$



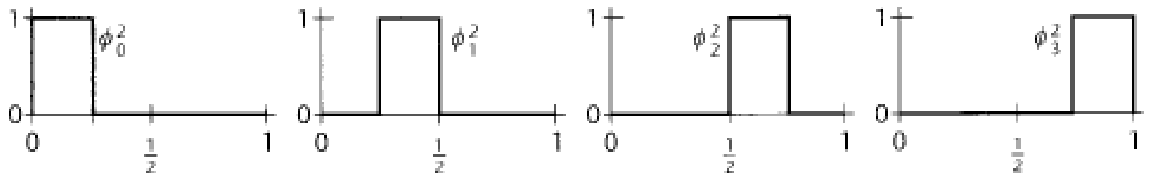


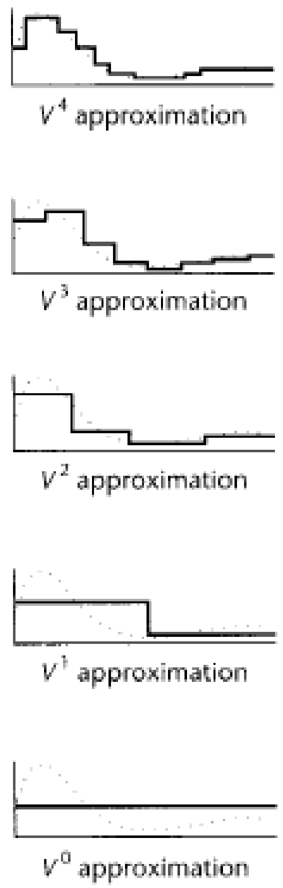
base for Haar - > square waves.

Coefficients *c*

$$f(x) = c_0 + \sum_{j=0}^{\infty} \sum_{k=0}^{2^j - 1} c_{jk}\, \psi_{jk}(x).$$

http://mathworld.wolfram.com/HaarFunction.html





EXEMPLO: signal

coefficients

$$f = \begin{pmatrix} 9 \\ 1 \\ 2 \\ 0 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix} + 4 \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} + 1 \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix},$$

$$\begin{pmatrix} 3 \\ 2 \\ 4 \\ 1 \end{pmatrix}$$

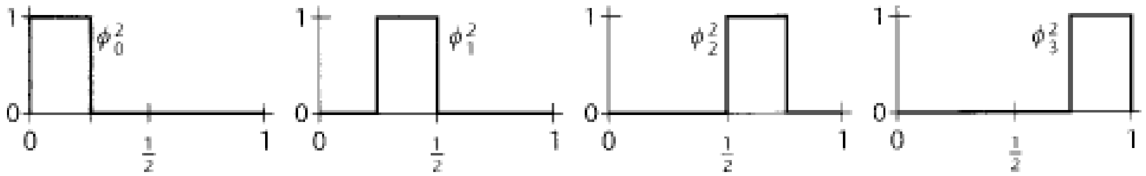
DC component - average

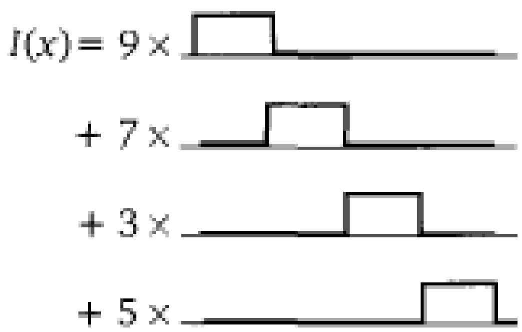
For 1 wave.

For 2 wavelets

<table>
<tr><td colspan="4">médias</td><td rowspan="2">R
e
s.
4</td><td colspan="2" rowspan="2">Deta-
lhes</td></tr>
<tr><td>9</td><td>1</td><td>2</td><td>0</td></tr>
<tr><td colspan="2">5</td><td colspan="2">1</td><td>2</td><td>4</td><td>1</td></tr>
<tr><td colspan="4">3</td><td>1</td><td colspan="2">2</td></tr>
</table>

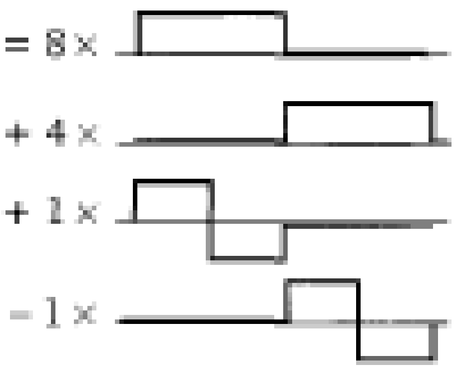
Exemplo: 9 7 3 5



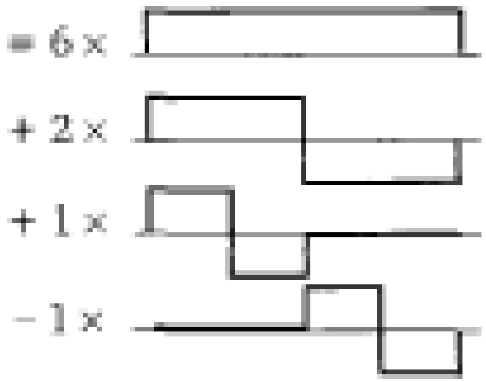


Using average and details:

$$I(x) = c_0^1\phi_0^1(x) + c_1^1\phi_1^1(x) + d_0^1\psi_0^1(x) + d_1^1\psi_1^1(x)$$



$$I(x) = c_0^0\phi_0^0(x) + d_0^0\psi_0^0(x) + d_0^1\psi_0^1(x) + d_1^1\psi_1^1(x)$$

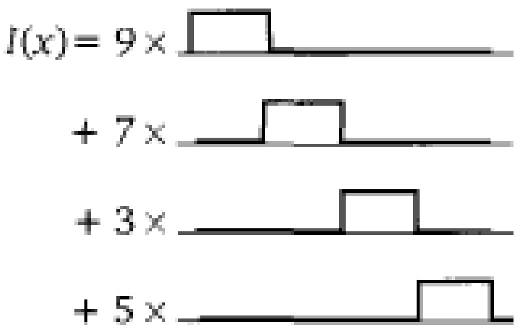


Resolution	Averages	Detail Coefficients
4	9 7 3 5	
2	8 4	1 −1
1	6	2

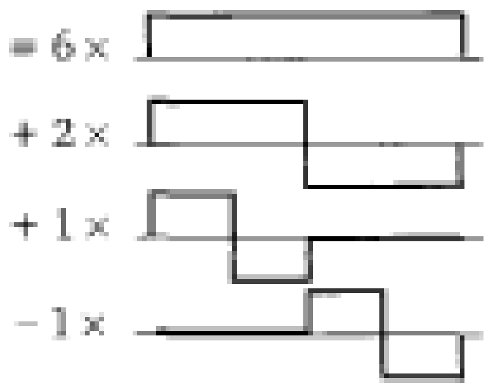
(this is multi resolution)

Lossless compression

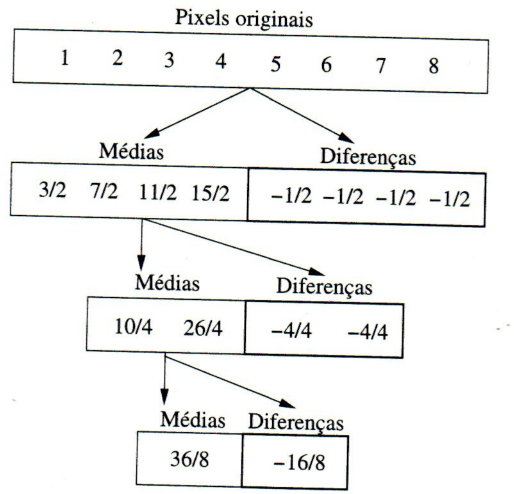
(11 bits x 7 bits)

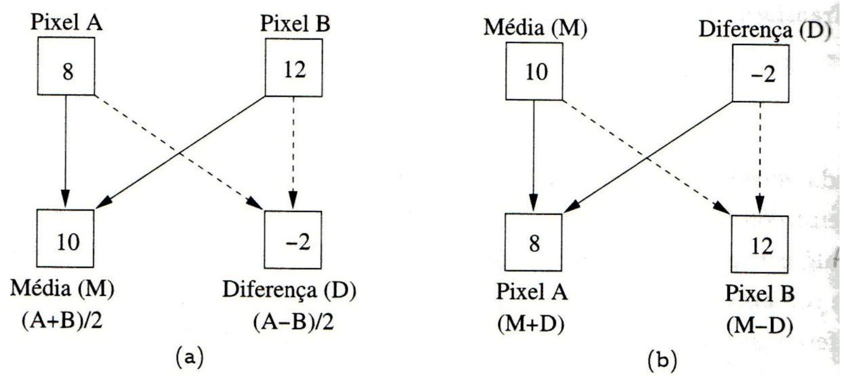


$$I(x) = c_0^0\phi_0^0(x) + d_0^0\psi_0^0(x) + d_0^1\psi_0^1(x) + d_1^1\psi_1^1(x)$$



Codification & recontruction





Normalized filters

$$\phi_i^j(x) := 2^{j/2}\phi(2^j x - i)$$
$$\psi_i^j(x) := 2^{j/2}\psi(2^j x - i)$$

- Normalized coefficients

$$6 \quad 2 \quad 1 \quad -1 \qquad\qquad 6 \quad 2 \quad \frac{1}{\sqrt{2}} \quad \frac{-1}{\sqrt{2}}$$

- New filters

$$l = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \qquad h = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$l = \begin{bmatrix} \frac{1+\sqrt{3}}{4\sqrt{2}} & \frac{3+\sqrt{3}}{4\sqrt{2}} & \frac{3-\sqrt{3}}{4\sqrt{2}} & \frac{1-\sqrt{3}}{4\sqrt{2}} \end{bmatrix} \quad h = \begin{bmatrix} \frac{1-\sqrt{3}}{4\sqrt{2}} & -\frac{3-\sqrt{3}}{4\sqrt{2}} & \frac{3+\sqrt{3}}{4\sqrt{2}} & -\frac{1+\sqrt{3}}{4\sqrt{2}} \end{bmatrix}$$

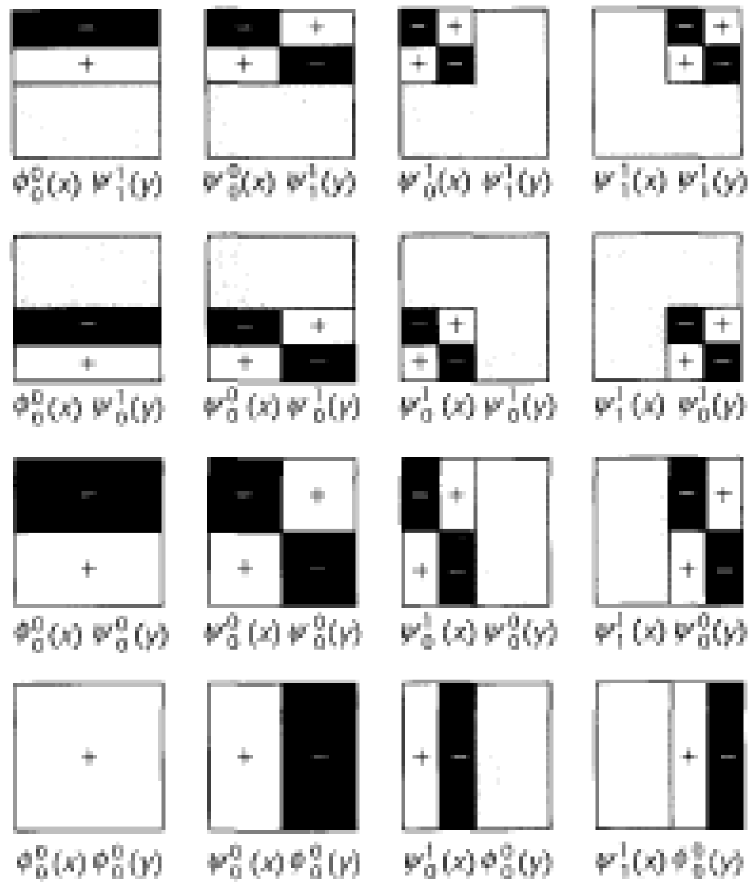
Haar 2D stantard **decomposition**

- using

  + = +1,

  - = -1 ,

  ■ =0

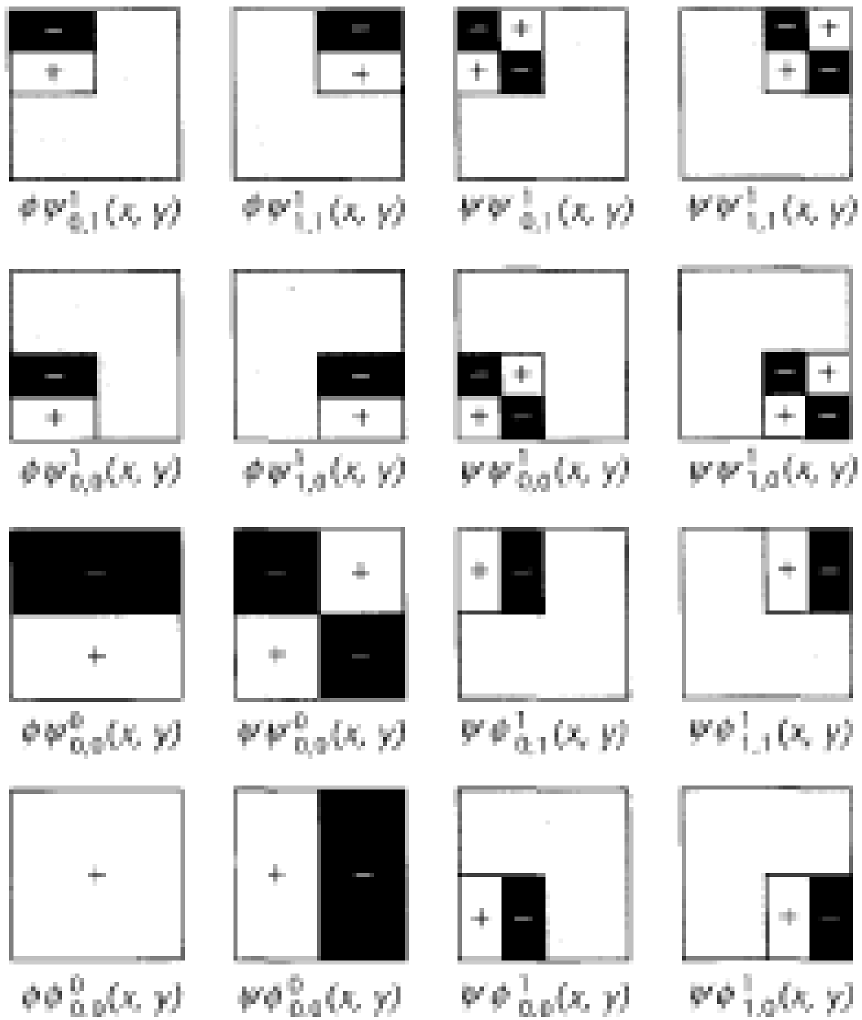


Haar 2D piramidal decomposition

– + = +1,

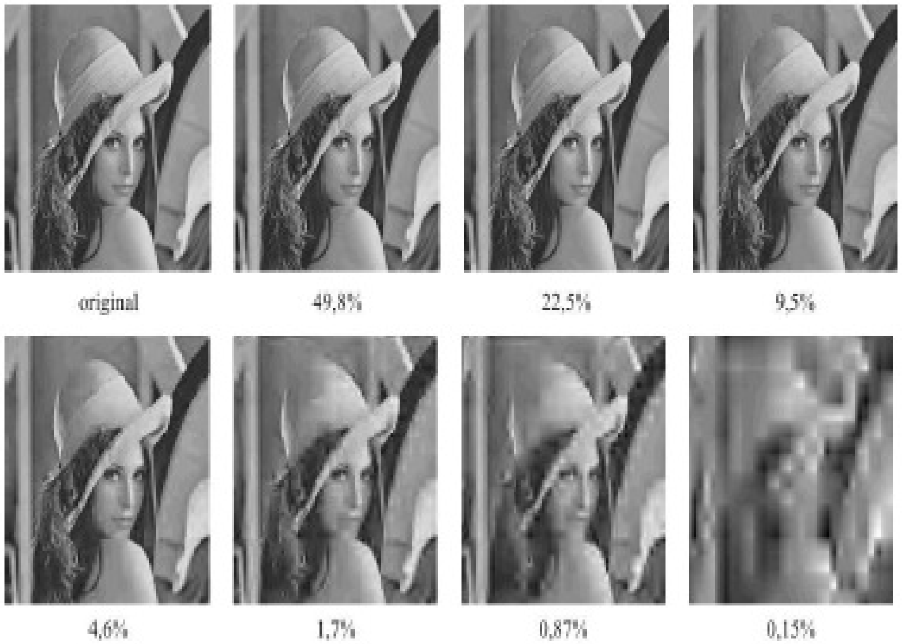
– - =-1 , e

=0



Thresholding

- Percentages of coefficients .
- with only 5% - > almost perfect reconstruction



usingDaub4

Wavelets and noise

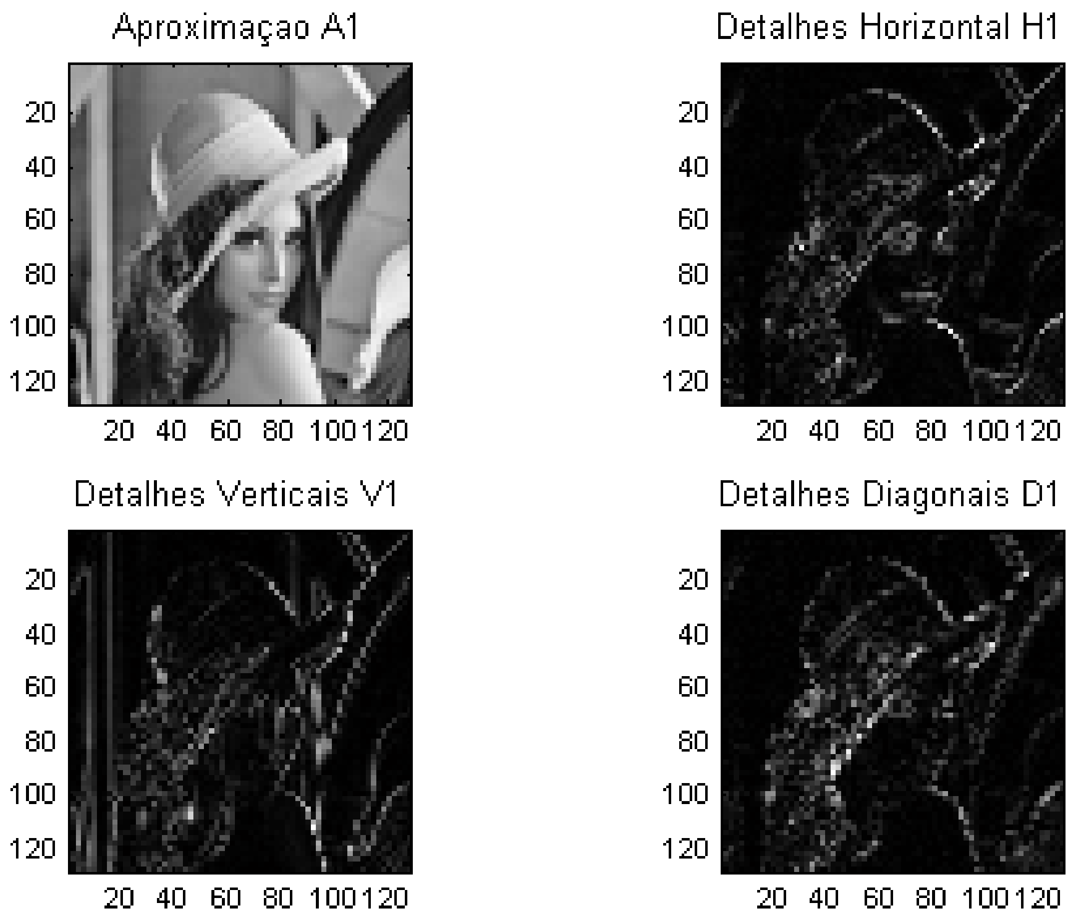
- *wavelet shrinkage ou thresholding*

performance

- Compression ratio

- Quality:
  - Root Mean Square Error (RMSE),
  - Sign Noise Ratio (SNR) and
  - Peak Sign Noise Ratio (PSNR)

coeficientes de detalhes



Onde está o ruido, na região suave ou nos detalhes?